Political Persuasion

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Abstract

How can competing political parties use persuasion to win elections? To study the role of competition in persuasion, I construct a voting model where two political parties compete by designing campaigns that release information about their party's candidate. By designing the whole campaign–as compared to designing a particular message–political parties are able to systematically change the beliefs of a Bayesian voter. Campaigns generate distributions of voter beliefs about the candidate's quality. Under competition, each party must worry about the other party and does not want to design a campaign that is easy to beat. In the unique equilibrium, both parties design campaigns that generate uniform distributions. The uniform distribution means that the voter is equally likely to have a range of beliefs about the candidate's quality after the campaign. It also means that the voter has maximum uncertainty about the candidates. The uniform distribution is a common feature in zero-sum games where each party wants to win but only needs a small margin, such as all-pay auctions or Colonel Blotto games. The paper also highlights a similarity between buying votes with money and using persuasion.

JEL-Classification: D72 (Political Processes), D82 (Asymmetric and Private Information), D83 (Information and Knowledge, Communication, Belief)
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Voice is political action par excellence.

Albert Hirschman 1970

1 Introduction

Political campaigns are filled with talking: at rallies, in commercials, or in debates. If there is any information in all that talking, political parties will want to use information to win. They can talk their way to victory in a combination of two ways. First, a party can convince voters that their candidate will be fantastic at doing what the voter wants. Second, a party can convince voters that their candidate is not as bad as the other party's candidate. The optimal strategy depends on what voters initially believe about both candidates and the other party's campaign strategy.

To better understand how politicians convey information to voters and how the information depends on the competition inherent in political campaigns, I construct a game of information disclosure with two senders and one receiver. The senders are political parties–Left Party and Right Party–and the receiver is a median voter. First in the game, each party decides what information to reveal about its candidate. In practice, this is like committing to a campaign schedule: parties pick debate days, choose the primary schedule, etc. After committing to a campaign schedule, specific information is revealed about the candidate. How the voter perceives the information is beyond the direct control of the party after each party has set up their campaign. The news media reports what comes out and the voter infers something about the candidates. Lastly, the voter uses her information and picks the winner. I model information disclosure formally as a Bayesian persuasion game (Kamenica and Gentzkow 2011).

Since there are two senders competing, each sender has to consider (1) how to influence directly the beliefs of the receiver and (2) the actions of the other sender. For each party, their campaign strategy is equivalent to choosing a distribution of voter beliefs. More than that, the game is zero-sum since any gain that one party generates from persuasion is a direct loss to the other party. I will sometimes referred to one party as the *maximizer* (Left) and the other as the *minimizer* (Right). The maximizer chooses a campaign strategy to maximize the probability that it wins.[1] The minimizer chooses a campaign strategy to minimize the probability that it loses.

The main result of the paper characterizes the unique equilibrium (Theorem 1). In equilibrium, each party picks a campaign that distributes the voter's beliefs uniformly. By uniform, I mean that when the voter is choosing the winner after the campaign, the voter is equally likely to believe the candidate is good with a probability of $0.010, 0.011, 0.011$, etc, and everything in between. The uniform distribution, while appearing strange, makes sense after a closer look. First, it cannot be that in equilibrium either party chooses a campaign strategy that generates positive mass on certain beliefs (Lemma 1). If the the Right Party generates positive mass on some point, the Left Party can beat that mass of beliefs by ϵ and increase its probability of winning discontinuously. Winning by ϵ means convincing the voter that your candidate is better than the other candidate, even if both candidates are bad. The only place this argument does not hold is at the highest possible belief, since there is no ϵ above that (Lemma 2). If the distribution of beliefs about the Right Party's candidate is continuous, the Left Party does not have such an easy target. Therefore, it must be that in equilibrium strategies are continuous distributions almost everywhere.

It is not just discontinuities that the Right Party will guard against. The Left Party benefits from persuasion if the Right Party's strategy is not concave. This *concavification* argument comes directly from Kamenica and Gentzkow (2011). If the Right Party's strategy is not concave, the Left Party can "concavify" and win with a higher probability. The Right

1. Kamenica and Gentzkow (2011) show that the maximized probability is the concave closure of the probability of winning for the realized beliefs. The geometry comes from Aumann and Maschler (1995, 128)

Party wants to prevent that option by picking a strategy that is already concave or close to concave. The uniform distribution is concave. A uniform strategy eliminates the ability of the Left Party to win by ϵ or concavify.

The uniform strategy closely resembles–in some cases is identical to–political parties' strategies when they are trying to buy votes, such as in Myerson (1993) and Sahuguet and Persico (2006). In those papers, if the Right Party gives \$1 to a positive mass of the population, the Left Party could give that same group \$1 + ϵ and win all their votes. In equilibrium, the distribution of payments to voters is uniform. Besides the equilibrium, another aspect of the vote-buying literature is worth mentioning. In that literature, the parties commit to paying out campaign promises, even though they may want to renege once in office. My paper, by modeling information disclosure as Bayesian persuasion, also assumes that the parties can commit to their campaign schedule. The identical commitment assumption allows me to draw a direct parallel between vote-buying and persuasion.

The paper highlights a similarity between the concavification argument of Aumann and Maschler (1995) and Kamenica and Gentzkow (2011) and the uniform strategies used in zero-sum games. The similarity should not be surprising. Aumann and Maschler (1995) derive their original concavification result in a repeated zero-sum game. Also, uniform strategies are concave for both players. While political competition through information disclosure is relatively non-standard, the equilibrium strategies turn out similar to the more standard models.

The rest of the paper is organized as follows. Section 2 goes through an example to relate concavification to uniform strategies. Section 3 lays out the formal model. I then turn the information disclosure game, which is between two senders and a receiver, into a zero-sum game between the two senders. I then derive the main theorem. Section 4 extends the equilibrium result to more general cases by harnessing concavification and zero-sum game results. Section 5 discusses related literature. Section 6 concludes.

2 Example

Consider a single median voter and two political parties, creatively named the Left Party and Right Party. The voter must decide whether to vote for the Left or Right Party. Each party is made up of two types of politicians. Call them Moderates and Extremists. For this example, half of each party is Moderate, half is Extreme, and this is common knowledge. The Moderates are good from the voter's perspective, because they share the preference of voter at 0. Left and Right Extremists are equally bad from the voter's perspective, because they prefer a policy that is distance 1 from the voter's. Once elected, a politician will implement his ideal policy. Placing the candidates and voter at their ideal policy, we have Figure 1. However, the voter does not directly see the politicians' types, but must rely on information disclosed by the political parties.

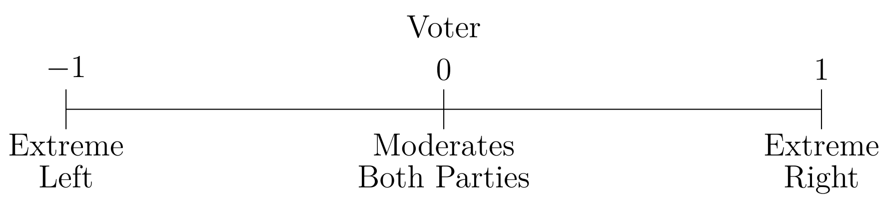

Figure 1: Political Spectrum

Parties only want to win the election. Each party maximizes the probability that their candidate wins. If parties could control who became their candidate, they would always choose a Moderate. Each party picks their candidate at random from their ranks, again made up of half Moderates and half Extremists. There is no strategic picking of candidates in order to isolate the role of information.

Each party can only influence the voter by choosing a *campaign schedule*. Before voting, the voter sees a realization of the signal, s, about the type of the candidate before the voter votes. The party chooses the distribution of the signal according to the candidate they have. Since this signal changes the voter's beliefs about the candidate, the party can affect beliefs by changing the distribution of the signal. Choosing the distributions is what I mean by designing a campaign that will send information to the voter. Note that even when the campaign is designed to make the candidate appear in some way to the voter, the perception that the voter gets cannot be perfectly controlled by the party. The party does not directly control s. More formally, a campaign for party i is a distribution $\pi_i(s|Moderate)$ and $\pi_i(s|Extreme)$. One can think of the campaign schedule as choosing how many ads to run, debates to have, etc. The voter updates her beliefs and attaches probability ℓ that the Left candidate is moderate and probability r that the Right candidate is moderate. If $\ell > r$, the voter picks the Left candidate and vice versa. If she is indifferent, she flips a coin.

Parties choose campaign schedules to manipulate the voter's beliefs about whether a candidate is moderate or extreme. In a campaign, each party can design π to induce a distribution of beliefs about the probability that their candidate is moderate, as long as the expectation equals the prior probability, that is $\mathbb{E}[\ell] = \ell_0 = 1/2$ and $\mathbb{E}[r] = r_0 = 1/2$. This follows from Bayes' Rule.

One campaign is where $\pi(s|Moderate) = \pi(s|Extreme)$. This campaign is completely uninformative. Seeing s provides the voter with no new information, since s is equally likely given a Moderate or Extreme candidate. The voter will simply maintain her prior belief. Another campaign is where the campaign fully reveals each candidate's type. However, neither of the strategies will be used in equilibrium. There is not even a best-response for the other party to these strategies, because of the discontinuity of the payoffs inherent in this voting model. Each party wants to win, but only needs to win by a small amount. Whatever margin they are winning by, they could to win by a smaller margin but more often.

To see that there is no best-response, suppose the Right Party is not revealing anything; $\pi_R(s|Moderate) = \pi_R(s|Extreme)$. Let R_U (for uninformative signal) be the CDF of posterior beliefs about the Right Party generated by this π_R. This is shown in the left half of Figure 2. With no new information, the voter believes the Right candidate is moderate with probability equal to 1/2. Given this π, which induces R_U, the Left Party win with probability one anytime its signal realization induces a belief $\ell > 1/2$. If $\ell = 1/2$. the voter flips a coin, so the Left Party wins with probability equal to one half. If $\ell < 1/2$, the Left Party loses. The right half of Figure 2 gives the probability the Left Party wins for any realized posterior belief.

Figure 2 shows the mapping from the CDF of one party (Right) to the probability of winning of the other party (Left) for the example. More generally, whatever belief the Left Party induces, ℓ, will beat the Right Party every time $\ell > r$, which occurs with a probability equal to the CDF R_U evaluated at ℓ. The only additional complication comes when $\ell = r$, where I assume the voter flips a coin.[2] Given the R_U, the Left Party chooses a campaign, π_L, to induce a distribution of posteriors about its own candidate so the the Left Party wins with the highest probability.

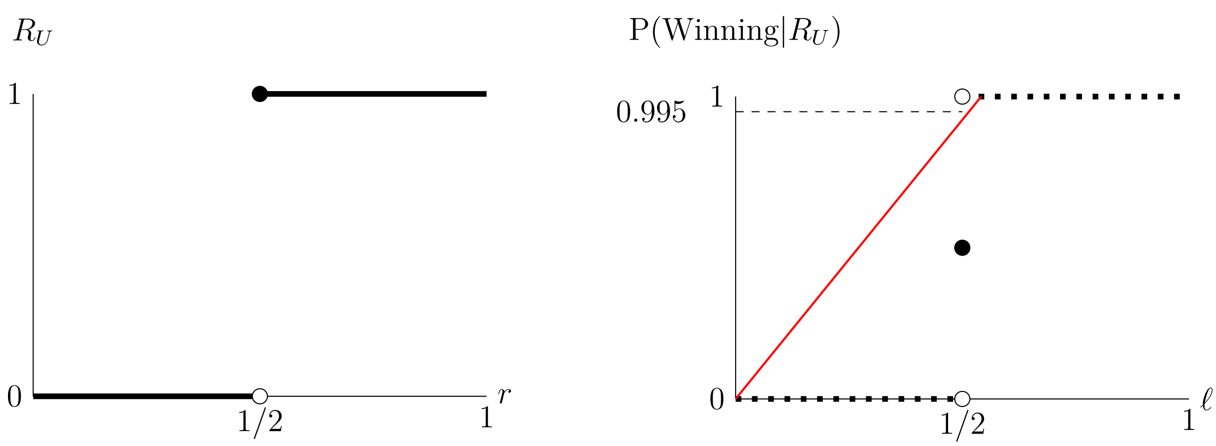

Figure 2: Right Plays Uninformative Strategy

However, the Left Party does not have a best-response to R_U. The Left Party only wants to induce a belief slightly above 1/2, subject to $\mathbb{E}[\ell] = 1/2$. Since it can always do better by winning by a smaller margin and therefore winning with a higher probability, the

2. The straightforward mapping from the CDF generated by the Right Party's strategy to the probability of winning for the Left Party remains throughout the paper. That is why I work with the constant-sum game, compared to a traditional zero-sum game.

best-response is undefined.

To see this, consider a strategy by the Left Party over $s \in \{M, E\}$:

$$\pi_L(M|Moderate) = 1 \qquad \pi_L(M|Extreme) = \frac{99}{100}$$
$$\pi_L(E|Moderate) = 0 \qquad \pi_L(E|Extreme) = \frac{1}{100}$$

With this strategy, the voter puts a probability strictly greater than 1/2 any time the signal M is realized, giving the election to the Left Party. The Left Party wins each time the Left Party is actually moderate and each time M is realized, even though the candidate is extreme. Therefore, the Left Party wins with a probability of $1/2 * 1 + 1/2 * (99/100) = 0.995$. That is a convex combination of the probability of winning at belief 0 (certainty that the politician is extreme is the voter sees E) and a belief greater than 1/2 when M is realized. The red line in the right half of Figure 2 gives the convex combination at $\ell_0 = 1/2$. However, this is not a best-response to R_U. The Left Party can always use a signal that generates a higher probability of winning, no matter what π_L is proposed. The Left Party can win with a probability arbitrarily close to one, but not one. If ties were broken in favor of the Left Party, as in Kamenica and Gentzkow (2011), then the Left Party could win with probability one. The Left Party could reach the concave closure of the probability of winning. With two senders, there is no justification for the voter to break ties in favor of one party and therefore the Left Party cannot always reach the concave closure, but only arbitrarily close.

However, it is not simply that the Left Party does not have a best-response. Since the game is zero-sum, the Right Party would benefit by "smoothing out" that discontinuity in the probability that the Left Party wins. Intuitively, the Right Party wants to choose the distribution of r in order to prevent the Left Party from winning by ϵ. If $S = [0, 1]$, the Right Party is able to completely smooth out the distribution of r and therefore the probability that the Left Party wins (and the Right Party loses) for any belief of ℓ.

Equilibrium requires that both parties smooth out their posterior distribution. The *unique* equilibrium of this example occurs when each party uses a set of signals to induce a uniform distribution of ℓ and r on [0,1], shown in Figure 3. First, notice that this is a Bayes-plausible distribution for the both parties, $\mathbb{E}[\ell] = 1/2 = \mathbb{E}[r]$; the parties can generate this distribution of posterior beliefs. To see that the uniform strategies make up an equilibrium, suppose that the Right Party is playing the uniform strategy, call the CDF $R^*(r)$. Then for the Left Party, the probability of winning for a certain induced posterior–against the uniform strategy–is also a straight line. Given any posterior, ℓ, the Left Party wins against all the realizations of r that are less than ℓ. This is exactly the CDF of R^* at any ℓ.

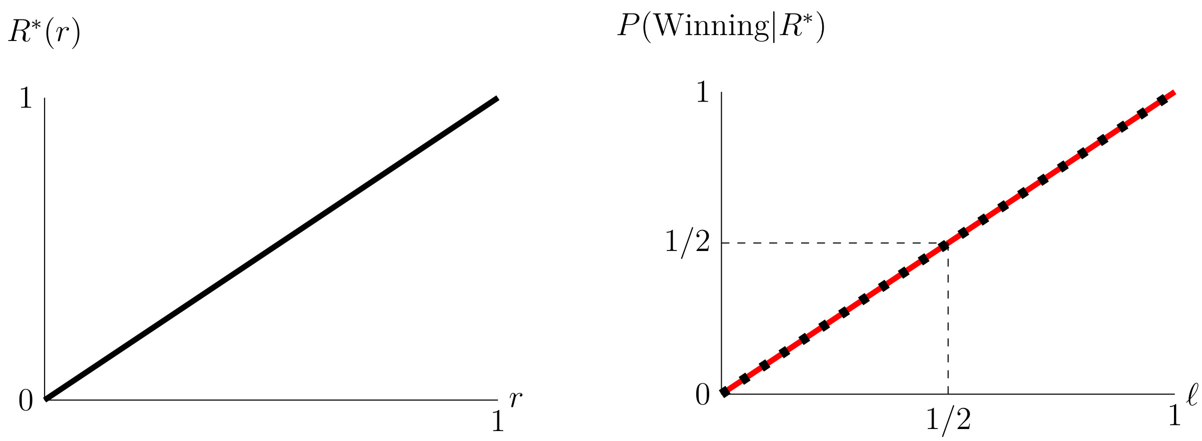

Figure 3: CDF to Probability of Winning

From Kamenica and Gentzkow (2011), the best that the Left Party can do is the *concavification* of P(Winning$|R_U$).[3] In Figure 3, the P(Winning$|R_U$) is the dotted line and the concavification is the red line. They exactly coincide. The Right Party eliminates the possibility of the Left Party benefiting from concavification. Therefore, the best the Left Party can do against the uniform strategy of the Right Party leads to winning half of the time. One strategy that ensures winning with probability 1/2 is the same uniform strategy as the Right Party. Therefore, by symmetry, the uniform strategy is a part of an equilibrium. That

3. The concavification of a function f is the smallest concave function that is everywhere as great as f.

each party wins with probability 1/2 comes from symmetry and is not a general feature of the game. Theorem 1 proves that uniform strategies make up the unique equilibrium more formally and for a larger class of games.

The uniform distribution comes from the concavification argument. Any failure of concavity in R, such as when R is discontinuous, can be concavified by the Left Party. The concavification benefits the Left Party and since the game is zero-sum hurts the Right Party. Instead of giving that advantage to the Left Party, the Right Party is able to remove any ability to concavify by using the uniform strategy. The next section extends the symmetric example to any prior probabilities.

3 Model

There are two political parties, indexed by $i \in \{\text{L, R}\}$, who receive a utility of 1 if they win the election, 0 if they lose, and have expected utility. Since a win for one party is a loss for the other, the game between the parties (ignoring the voter) is constant-sum, which is equivalent to a zero-sum game. The value of the game is the probability that the Left Party wins. Label the Right Party as the minimizer and the Left Party as the maximizer. There is a single voter (she) who chooses the winning party. Instead of uncertainty about a candidate's policy position, as in the example, assume there is uncertainty about whether each party's candidate will be good for the voter, i.e. the quality of the candidate. Each candidate can either be bad or good quality for the voter, $q \in \{0, 1\}$. If the candidate is bad for the voter, the voter receives a utility of zero. If the candidate is good for the voter, the voter receives a utility of one. She also has expected utility[4]

The parties and voter have a shared prior belief about whether each party's candidate will be good. For the Left Party, the candidate is good with a probability of $\ell_0 \in (0, 1)$. For

4. The restriction to two types of policies for each party is not without loss. See Gentzkow and Kamenica (2016a). This is elaborated on in Section 4.

the Right Party, the probability is $r_0 \in (0,1)$. These realizations of candidate quality are independent across the parties. Instead of picking policies or candidates strategically, the parties compete by designing campaigns to release information about the quality of their candidate.

Each party's strategy is a campaign and denoted by π_i. A campaign is made up of signal realizations, $s_i \in S = [0,1]$, and a set of conditional distributions, $\{\pi_i(s_i|q)_{q\in\{0,1\}}\}$.[5] Each party chooses its campaign without knowing the signal realization of the campaign. The voter updates her beliefs according to Bayes' Rule to a posterior belief about the probability that the candidate is good. Denote a particular pair of realized posteriors $(q_L, q_R) \in [0,1]^2$. From any belief, the voter can calculate the expected candidate quality. Conditional on her beliefs, the voter chooses a probability of voting for the Left Party, denoted by $v(q_L, q_R)$, the value of the corresponding zero-sum game.

Instead of working directly with the set of campaigns, it is more convenient to consider distributions of posterior beliefs that a party chooses to induce. Formally, any strategy π_i and a realization s_i induces a particular posterior belief $q_L \in [0,1]$ and $q_R \in [0,1]$. Before knowing any particular realization, any strategy generates a distribution of posteriors. Let $L(q_L)$ and $R(q_R)$ be the CDF of beliefs induced. The only restriction that Bayes' Rule imposes on the posteriors is that the expected posterior equals the prior:

$$\textstyle\int_0^1 q_L dL(q_L) = \ell_0, \qquad \int_0^1 q_R dR(q_R) = r_0.$$

Any distribution of posteriors satisfying the above condition is called *Bayes-plausible*. Instead of the parties choosing π_i, it is as if the party chooses a Bayes-plausible distribution of posteriors. A direct strategy in this game has the parties telling the voter what belief to have, compared to more standard direct strategies where the receiver is told what action to

5. Kamenica and Gentzkow (2011) assume that S is finite. As Lemma 1 shows, because each party's payoff is discontinuous, there will be no equilibrium when S is finite. I need to allow for a larger signal space to smooth out the discontinuities.

take. In this formulation, the set of feasible strategies for a party is the set of Bayes-plausible distribution of posteriors. Equilibrium requires that each party chooses a distribution of posteriors that maximizes the probability of winning the election, given the other party's strategy and the voter's strategy.

Equilibrium: The equilibrium concept I use is *fair subgame perfect equilibrium*[6]. Equilibrium is a strategy of the Left Party, $L(q_L)$, Right Party, $R(q_R)$, and the voter, $v(q_L, q_R)$, that satisfies four conditions:

I. given Right Party's and the voter's strategies, the Left Party chooses a Bayes-plausible distribution to maximize $\mathbb{E}[v(q_L, q_R)]$;

II. given the Left Party's and the voter's strategy, the Right Party chooses a Bayes-plausible distribution to minimize $\mathbb{E}[v(q_L, q_R)]$;

III. the voter chooses the candidate with the highest expected quality;

IV. if the voter is indifferent, she votes based on the toss of a fair coin.

The fourth condition is why I refer to the equilibrium as *fair*. Focusing on fair subgame perfect equilibria allows us to directly talk about the probability that the Left Party wins given any posterior beliefs.[7] That is, in any fair subgame perfect equilibrium

$$v^*(q_L, q_R) = \text{Equilibrium Probability Left Party Wins} = \begin{cases} 1 & q_L > q_R \\ \frac{1}{2} & q_L = q_R \\ 0 & q_L < q_R. \end{cases}$$

6. Kamenica and Gentzkow (2011) use *sender-preferred* subgame perfect equilibrium which means that when the receiver is indifferent, she picks what the sender wants. Since there are two senders (the two parties), their justification does not apply. That solution concept also avoids problems generated by the discontinuity in the motivating example. With two senders, this cannot apply.

7. If we did not restrict attention to fair subgame perfect equilibria, the voter's decision would not be pinned down by beliefs, but instead it would be an equilibrium object.

The probability that the Right Party wins is $1 - v^*(q_L, q_R)$.[8] The Left Party selects its campaign to maximize $\mathbb{E}[v^*(q_L, q_R)]$. The Right Party minimizes $\mathbb{E}[v^*(q_L, q_R)]$. However, this may not be straightforward. Unlike in Kamenica and Gentzkow (2011), $v^*(q_L, q_R)$ is *not* upper-semi continuous. Therefore, a best-response is not defined for every strategy of the other player, as Lemma 1 shows. Call a strategy *uninformative* if it puts probability one on the prior belief.

Lemma 1. *Fix the strategy for one party that uses finite set of posteriors. If the other party wins with probability one by using an uninformative signal, the uninformative signal is a best-response. Otherwise, there is no best-response.*

Proof. Consider the best-response of the Left Party. Fix a strategy of the Right Party that uses a finite set of posteriors; call it $R(q_R)$. That strategy is a step function and is given on the left half of Figure 4. The strategy $R(q_R)$ generates a $v^*(q_L; R(q_R))$, such as the black line in right half of Figure 4.

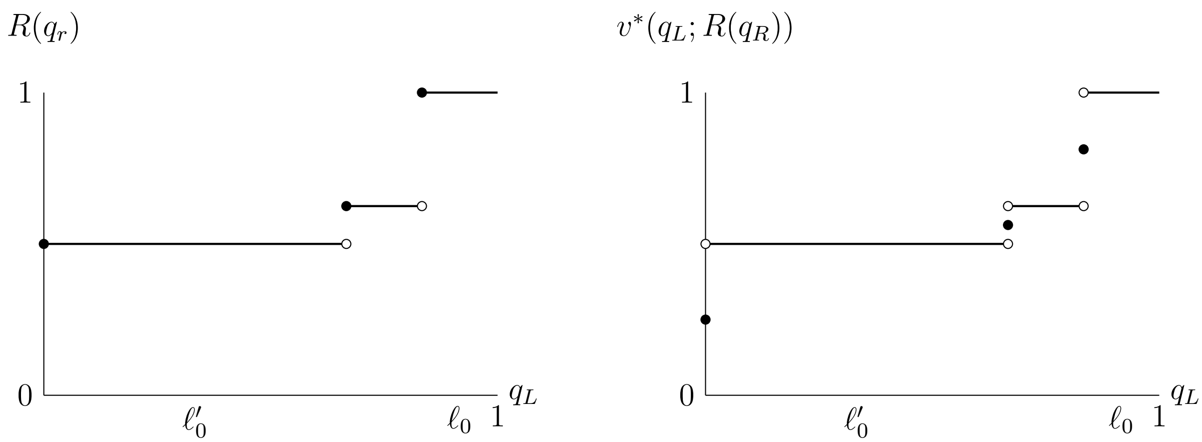

Figure 4: Finite Signal Realizations

First, consider ℓ_0 that is high enough that an uninformative signal is above any belief that the Right Party generates. The uninformative signal is a best-response, leading to the

8. The asterisk is to denote that I am looking at fair subgame perfect equilibrium. Notice that in other equilibrium concepts, the voter need to flip a coin. She can do any action when $q_L = q_R$.

the Left Party winning with probability one, i.e. $v^*(\ell_0; R(q_R)) = 1$.

Next, take any ℓ_0' where $v^*(\ell_0'; R(q_R)) < 1$. The Left Party is able to get any convex combination of $v^*(q_L; R(r))$ at ℓ_0' as Kamenica and Gentzkow (2011) show. However, since $v^*(q_L; R(q_R))$ is not upper-semicontinuous, no matter what combination used, there exists a feasible strategy for the Left Party that can achieve a higher probability of winning. Therefore, there is no best-response for the Left Party. ■

Lemma 1 rules out a whole class of strategies from ever being part of an equilibrium. Even though there is no best-response for *some* strategies of the other party, there is a best response to other strategies. If the Right Party chooses a continuous distribution, then the value function for the Left Party is continuous and a best-response does exist. Lemma 2 formalizes this.

Lemma 2. *Fix the strategy for one party that is continuous on* $[0, 1)$*. Then the other party has a best-response, although it need not be unique. Any best-response achieves the concave closure of the other party's strategy.*

Proof. Fix a $R(q_R)$ that is continuous $[0, 1)$. Because $R(q_R)$ is a CDF, $R(q_R) \leq 1$ and $R(1) = 1$. This implies that $R(q_R)$ is upper-semicontinuous on $[0, 1]$. This implies that $v^*(q_L; R(q_R))$ is also upper-semicontinuous because of the mapping from $R(q_R)$ to $v^*(q_L; R(q_R))$. We can therefore invoke Proposition 1 in Kamenica and Gentzkow (2011), which proves that the optimal value is attained at the concave closure of $v^*(q_L; R(q_R))$ and it therefore exists. ■

If $R(q_R)$ is continuous, there is no longer a discontinuous increase in probability of winning at any induced belief. The Right Party effectively takes away the "easy" win by an arbitrarily small margin. With a best-response existing for some strategies, equilibria have a chance of existing, even though $v^*(q_L, q_R)$ is discontinuous. One does.

Fixing the strategy of the voter, the model becomes a two-player, zero-sum game where the voter generates the payoff function, $v^*(q_L, q_R)$. Again, the possible strategies for the players are any Bayes-plausible distributions.[9] The full strategy is characterized in Theorem 1, which is the main result of the paper.

Theorem 1. *Let $\ell_0 \geq r_0$. There exists a* unique *equilibrium. In equilibrium, the Left Party wins with probability* $1 - r_0/2\ell_0$.

If $\ell_0 \leq 1/2$, the strategies are

$$L^*(q_L) = \begin{cases} \frac{1}{2\ell_0} q_L & q_L \leq 2\ell_0 \\ 1 & q_L > 2\ell_0 \end{cases} \qquad R^*(q_R) = \begin{cases} \left(1 - \frac{r_0}{\ell_0}\right) + \left(\frac{r_0}{\ell_0}\right) \frac{1}{2\ell_0} q_R & q_R \leq 2\ell_0 \\ 1 & q_R > 2\ell_0. \end{cases} \tag{1}$$

If $\ell_0 > 1/2$, the strategies are

$$L^*(q_L) = \begin{cases} \frac{1}{2\ell_0} q_L & q_L \leq 2(1-\ell_0) \\ \frac{1}{\ell_0} - 1 & q_L \in (2(1-\ell_0), 1) \\ 1 & q_L = 1 \end{cases} \qquad R^*(q_R) = \begin{cases} \left(1 - \frac{r_0}{\ell_0}\right) + \left(\frac{r_0}{\ell_0}\right) \frac{1}{2\ell_0} q_R & q_R \leq 2(1-\ell_0) \\ \left(1 - \frac{r_0}{\ell_0}\right) + \left(\frac{r_0}{\ell_0}\right) \left(\frac{1}{\ell_0} - 1\right) & q_R \in (2(1-\ell_0), 1) \\ 1 & q_R = 1. \end{cases} \tag{2}$$

Proof. First consider the case where $\ell_0 \leq 1/2$. This includes the example from Section 2, where $\ell_0 = r_0 = 1/2$. Suppose the Right Party is using $R^*(q_R)$ from Equation (1). With continuous probability distributions, the probability that the Left Party wins is the probability

9. This simplification allows me to harness the zero-sum game literature to solve for equilibrium. In particular, the persuasion game becomes a variant of Colonel Blotto games, which Hart (2008) has named a continuous "General Lotto" game.

that it generates $q_L \geq q_R$:

$$\begin{aligned} P(q_L \geq q_R) &= \int_0^1 R^*(x)dL(x) \\ &= \int_0^1 \min\left\{ \left(1 - \frac{r_0}{\ell_0}\right) + \left(\frac{r_0}{\ell_0}\right) \frac{1}{2\ell_0} x, 1 \right\} dL(x) \\ &\leq \left(1 - \frac{r_0}{\ell_0}\right) + \left(\frac{r_0}{\ell_0}\right) \frac{1}{2\ell_0} \int_0^1 x dL(x) \\ &= \left(1 - \frac{r_0}{\ell_0}\right) + \left(\frac{r_0}{\ell_0}\right) \frac{1}{2\ell_0} \ell_0 \quad \text{(Bayes-plausible)} \\ &= 1 - \frac{r_0}{2\ell_0}. \end{aligned} \tag{3}$$

With $R^*(q_R)$, the Right Party guarantees that the Left Party wins with probability less than $1 - r_0/2\ell_0$. It minimizes the upper-bound on how much the Left Party can win. Geometrically, the uniform distribution minimizes the concavification. Therefore, the value is less than $1 - r_0/2\ell_0$.

Now assume the Left Party is following the given strategy. Do the same calculation as Equation 3 for $P(q_R \geq q_L)$ to find that it is less than or equal to $r_0/2\ell_0$. The Left Party guarantees that the Right Party wins with probability less than $r_0/2\ell_0$. Therefore, the value is greater than $1 - r_0/2\ell_0$.

By the Minimax Theorem, if the guaranteed payoff for the Left Party equals one minus the guaranteed payoff for the Right Party, there exist strategies to support it. Therefore, the $L^*(q_L)$ and $R^*(q_R)$ make up an equilibrium. This also implies that every equilibrium has the value generated by $L^*(q_L)$ and $R^*(q_R)$.

For uniqueness, recall that by Bayes' Theorem the integral of the CDF is related to the prior by,

$$\int_0^1 dL(q_L) = 1 - \ell_0. \tag{4}$$

Therefore, any feasible distribution must have the same integral. For a contradiction to uniqueness, suppose an alternative strategy for the Left Party $L'(q_L) \neq L^*(q_L)$ was part of an equilibrium. First suppose $L'(q_L) < L^*(q_L)$ at a $q'_L \leq \ell_0$. By Equation 4, there must be an alternative point, $q''_L \geq \ell_0$, where $L'(q_L) > L^*(q_L)$. An example is given by the solid black line on the left half of Figure 5. The equilibrium strategy $L^*(q_L)$ is given by the dashed line for reference.

Given $L'(q_L)$, the probability that the Right Party wins for an induced belief q_R is $1 - v^*(q_R; L'(q_L))$. That function is the dotted line in the right half of Figure 5. The Right Party's best-response achieves the concavification at r_0 (Kamenica and Gentzkow 2011). The concavification is given by the red line. The value of the concavification at r_0 is strictly greater than the concavification against $L^*(q_L)$. That is, the Right Party could use concavification to win with a higher probability than equilibrium using strategies in Equation 1. Therefore, if $L'(q_L)$ was part of an equilibrium it would have a different value than the original equilibrium. This is a contraction for any $L'(q_L) < L^*(q_L)$, when $q'_L \leq \ell_0$. The same argument holds for $L'(q_L) > L^*(q_L)$ at a $q'_L \leq \ell_0$. Therefore, it must be that Equation 1 is the unique equilibrium for $\ell_0 \leq 1/2$.

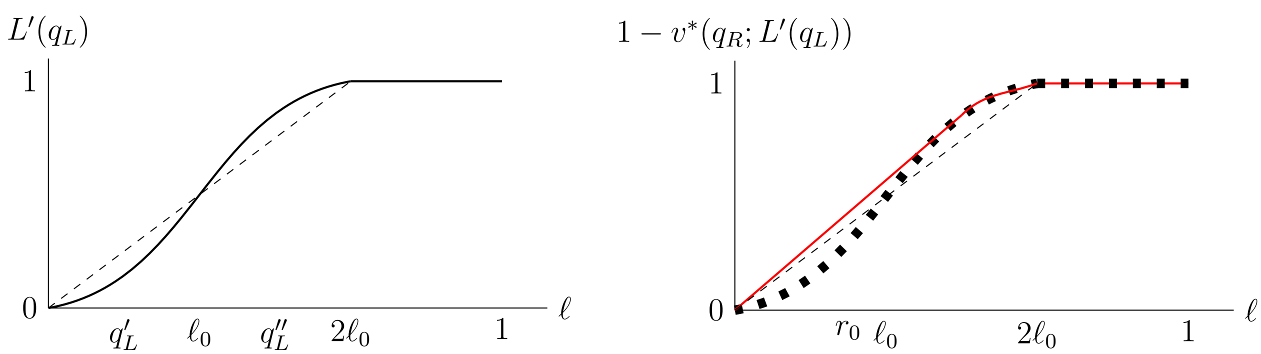

Figure 5: Equilibrium Uniqueness

For the cases where $\ell_0 > 1/2$, the uniform distribution is still used in equilibrium, but it is slightly changed. It has a flat part for $q_L \in (2(1 - \ell_0), 1)$. The flat part, while changing the shape of the distribution, does not change the algebra. Suppose the Right Party using the strategy in Equation 2:

$$
\begin{aligned}
P(q_L \geq q_R) &= \int_0^1 R^*(x) dL(x) \\
&= \int_0^1 \min\left\{ \left(1 - \frac{r_0}{\ell_0}\right) + \left(\frac{r_0}{\ell_0}\right) \frac{1}{2\ell_0} x, \left(1 - \frac{r_0}{\ell_0}\right) + \left(\frac{r_0}{\ell_0}\right) \left(\frac{1}{\ell_0} - 1\right) \right\} dL(x) \\
&\leq \left(1 - \frac{r_0}{\ell_0}\right) + \left(\frac{r_0}{\ell_0}\right) \frac{1}{2\ell_0} \int_0^1 x dL(x) \\
&= \left(1 - \frac{r_0}{\ell_0}\right) + \left(\frac{r_0}{\ell_0}\right) \frac{1}{2\ell_0} \ell_0 \quad \text{(Bayes-plausible)} \\
&= 1 - \frac{r_0}{2\ell_0}.
\end{aligned} \tag{5}
$$

Again, the Right Party has chosen a strategy to minimize the probability that the Left Party wins.

For uniqueness of the second case, again recall that for zero-sum games all equilibria have the same value. Also, note there exists no concave CDF that is Bayes-plausible when $\ell_0 > 1/2$, because Equation 4 must hold. However, with positive probability placed at $q_L = 1$, where the party cannot lose by ϵ, this is the only CDF that is Bayes-plausible and that the concave closure at r_0 that equals the value generate by Equation 2. ■

To connect the geometric Bayesian persuasion argument from the example with the optimal equilibrium strategies, consider the case where $\ell_0 = r_0 > 1/2$. Geometrically, the Left Party will concavify $v^*(q_L; R^*(q_R))$. The Right Party's strategy minimizes the concavification, given by the red line on the right at r_0 in Figure 6.

In fact, the Left Party's strategy minimizes the concavification for any prior probability of the Right Party. This leads to the following corollary.

Corollary 1.1. *Let $\ell_0 \geq r_0$. Then the Left Party's equilibrium strategy does not depend on the prior probability that the Right Party's candidate is good for the voter.*

Whichever party has a higher prior probability does not need to worry about the strength of the other party. The stronger party is in control of the election. The weaker party induces

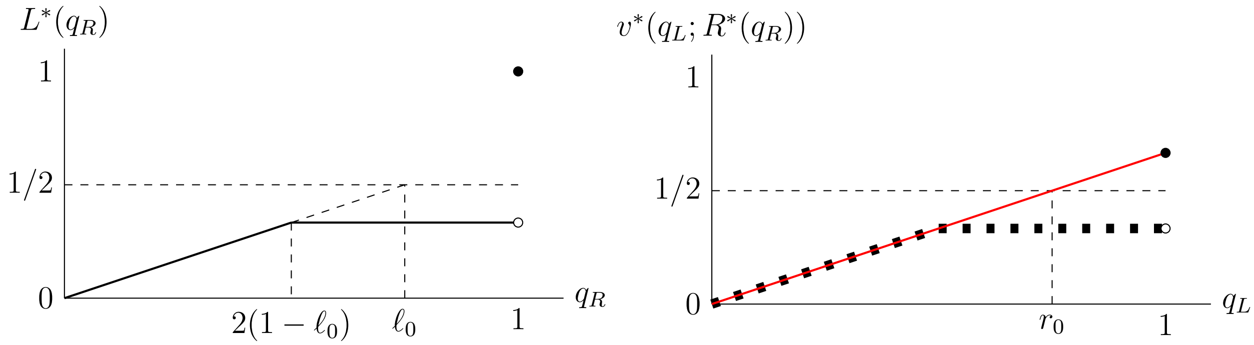

Figure 6: Concave Closure

a posterior belief of zero in an attempt to compete in the other elections.

In all cases, both parties use a partially uniform strategy. For the uniform part of the distribution, each political party designs their strategy to give away as little information as possible. Since any information that the Right Party could give away with its strategy will be used against it by the Left Party, it wants to avoid this. This also means giving as little information to the voter as possible. The uniform distribution does this exactly. One way to see this is to consider the entropy of the distribution chosen. A higher entropy means higher uncertainty.[10] For random variables on [0,1], the uniform distribution is the maximum entropy distribution. It has the highest uncertainty. For the voting example, the uniform distribution of beliefs means that the other party knows as little as possible about the other party's induced beliefs. Each party generates the most uncertainty it can. Caught in the middle is the voter.

The only part of the strategy that has positive mass occurs at zero or one. The mass at one is used because there is a cap on the posterior beliefs, which cannot be above one. For the mass at zero, consider the choice of the Right Party. The Right Party, which has a lower prior probability having a good candidate, cannot compete in every election. They throw some elections by inducing a belief a zero with probability $1 - r_0/\ell_0$. Since the Left Party

10. Cover and Thomas (2006, 6) state that "entropy is the uncertainty of a single random variable."

never induces a belief of zero, the Right Party loses those elections with certainty. However, by strategically throwing some elections, the Right Party is able to compete in the other elections, which occur with probability r_0/ℓ_0. In those remaining elections, the Right Party follows a uniform like the Left Party.

4 Beyond Bad and Good Types

Much of the simplicity of the strategies comes from there being two types of politicians for each party: bad and good. Instead, suppose the voter has preferences defined over the candidate quality, $q \in [0, 1]$, given by $u(q) = -|1-q|$. A candidate of type $q = 0$ is the worst and $q = 1$ is the best. Again, suppose everyone has expected utility. The parties start with a prior probability distribution over $[0, 1]$. Let $L(q)$ and $R(q)$ be the prior CDF for the Left Party and Right Party. It is easier to drop the subscript for the remaining discussion.

When there are more than two types the nature of the signal changes. The parties no longer want to send a signal about the probability the candidate is good, but the expected quality of the candidate. Denote the prior expectations by $\mathbb{E}_L[q]$ and $\mathbb{E}_R[q]$. The parties choose distributions over expected quality for their strategies. However, as elaborated in Gentzkow and Kamenica (2016a), not every distribution of expected quality that equals the prior expected quality can be generated.[11] However, in some cases, there exists a direct analog strategy from the two-type problem is feasible for the general case. By analog strategy, I mean ℓ_0 is replaced by $\mathbb{E}_L[q]$ and r_0 is replaced by $\mathbb{E}_R[q]$ in Equations 1 and 2. Define the strategies to be $L^*(q)$ and $R^*(q)$. The next two corollaries consider situations when we can directly apply the result of Theorem 1.

The first of these corollaries gives knife-edge cases where the strategy follows exactly from

11. A simple example should highlight the problem. Suppose there are three types of candidates denoted by their quality, $q \in \{0, 1/2, 1\}$, and they each occur with equal probability. The prior expected position is $1/2$. However, it is impossible to generate a distribution of posterior beliefs with probability $1/2$ both at 0 and 1.

the two-type case.

Corollary 1.2. *Suppose the prior CDFs for both parties are given by $L^*(q)$ and $R^*(q)$ from Theorem 1. Then there exists an equilibrium where both parties use fully-informative strategies.*

The proof follows immediately from Theorem 1 and the observation that a fully-informative strategy is always feasible. Note that Corollary 1.2 does not allow fully-revealing strategies in an equilibrium with *finite* types. In that case, Lemma 1 still applies and establishes that there is no best-response to a signal with a finite number of realizations.

While Corollary 1.2 shows a knife-edge case where fully-informative signals are used, the equilibrium strategies from Theorem 1 apply to more general distributions, although they are not fully-informative. To say more about the cases where equilibrium is similar in structure to the two cases, I adopt notation from Gentzkow and Kamenica (2016a). Define $c_X(q)$ to be the integral of a CDF X, i.e.

$$c_X(q) = \int_0^q X(t)dt.$$

The X could be the Left Party's or Right Party's prior or posterior distribution. In addition, let $c_{\overline{\pi}}$ be the integral of a fully-informative signal and $c_{\underline{\pi}}$ be the integral of an uninformative signal. The following result from Gentzkow and Kamenica (2016a) explains which posterior distributions of the expected quality are feasible.

Result 1 (Gentzkow and Kamenica (2016a))**.** *Given any convex function $c : [0,1] \to \mathbb{R}$ such that $c_{\overline{\pi}} \geq c \geq c_{\underline{\pi}}$, there exists a signal that induces it.*

Given the set of feasible distributions of expected quality, one can simply check whether the proposed distribution strategy is feasible using Result 1. The relevant inequality to check is whether $c_{\overline{\pi}} \geq c$, since the other inequality is satisfied for any distribution of expected quality where the expectation equals the prior. Corollary 1.3 incorporates this new constraint to clarify when the equilibrium strategies of Theorem 1 extend to the more general case.

Corollary 1.3. *Let $\mathbb{E}_L[q] \geq \mathbb{E}_R[q]$. Then for every prior distributions $L(q)$ and $R(q)$ such that the integrals of the fully-informative signal are both greater pairwise than the corresponding integrals of $L^*(q)$ and $R^*(q)$, i.e. $c_L[q] \geq c_{L^*}[q]$ and $c_R[q] \geq c_{R^*}[q]$ for every $q \in [0,1]$, then $L^*(q)$ and $R^*(q)$ are the equilibrium strategies..*

Informally, the prior probability must put enough weight near zero. That ensures that the integral of the equilibrium strategy considered is less than the fully-informative signal and the constraint $c_{\bar{\pi}} \geq c$ is not violated. If that is true, the proposed equilibrium strategies from Theorem 1 are feasible. This fact shows how the uniform strategy is a more common result than for just the two-type case. Therefore, even though Result 1 complicates the equilibrium for some distributions, there are a class of distributions where Theorem 1 directly extends. However, in more general cases, the equilibrium strategies will not directly correspond to the two-type equilibrium. The problematic distribution in footnote 11 comes because the proposed strategy puts too much weight around $q = 0$ and $q = 1$ relative to the prior distribution and is therefore infeasible.

While theoretically clean and tractable, generating a uniform distribution of posteriors is not clear for real campaigns. Such a strategy requires a continuum of outcomes to the campaign. However, as Cover (1974) shows, any distribution of posteriors such that $\mathrm{E}[r] = 1/2$ can be generated by a sequence of fair coin tosses. To take the voting example, a uniform distribution of beliefs can be generated by a sequence of days of information. On one day, the party decides to release a big piece of news. There is a 50% chance that the voter interprets that news as coming from a moderate. On another day, the party releases "half" as much news. On a third, half of that. A campaign strategy of this type will generate a uniform distribution of posterior beliefs on [0,1], which is what each party wants.[12]. However, this

12. The formal argument comes from Bell and Cover (1980, p. 163) Suppose a gambler starts with 1/2

requires an infinite sequence of news-cycles. That is never actually possible, but maybe 597 days of campaigning for president is a close approximation. Maybe not. However, as said before, the uniform strategy is common in zero-sum games and widely used. The next section elaborates on some of these results and other related literature from voting and Bayesian persuasion.

5 Related Literature

The current paper is closest to the literature of elections that are model as zero-sum Colonel Blotto games (Gross and Wagner 1950; Roberson 2006). I have already mentioned that the closest voting papers are Myerson (1993) and Sahuguet and Persico (2006). In Myerson (1993), candidates compete by offering benefits to voters; the candidates are buying votes. As in this paper, a strategy is a distribution; the probability density is how many voters receive a certain level of benefits. The candidates have budget constraints which mimic the Bayes-plausible restriction in my paper. The equilibrium strategy is a uniform distribution of benefits.[13] As Roberson (2006) shows, the uniform distribution is a general result for Colonel Blotto games, where the players want to win part of the game, whether they be battlefields or voters, by as little as possible. In Myerson, each candidate wants to offer ϵ more than the other candidate. Sahuguet and Persico extend Myerson to include asymmetries between the parties. The uniform strategies in Equation (1) in Theorem 1 are the same strategies as in Theorem 1 in Sahuguet and Persico (2006, 104). Taking those papers and this paper together, there are two ways of "buying" votes: actual payments or beliefs. This paper focuses on beliefs and shows how political parties can go about "outbidding" the other party

unit of capital. He divides the capital into piles of $\frac{1}{2}\left(\frac{1}{2}\right)^i$, for $i = 1, 2, ...$ There is a sequence of fair coin flip and the gambler bets the ith pile on the ith coin flip. The expected outcome of the sequence of bets is distributed uniformly on [0,1].

13. Bell and Cover (1980) find the same equilibrium strategy for competing investors who have $1 to place on fair bets.

with beliefs.

The way I chose to model information manipulation comes from the Bayesian persuasion literature, started by Kamenica and Gentzkow (2011).[14] Three papers on Bayesian persuasion and information manipulation are especially worth mentioning and connecting to my paper. One because the setup is similar to my paper and the other two because they study elections.

Au and Kawai (2016) study a general form of competition in persuasion. Their main results apply to two competing senders with a finite set of proposals that a receiver must choose between. They similarly derive uniqueness of a uniform equilibrium. There are some differences though that are worth pointing out. They assume an outside option for the receiver. In the voting context, I do not believe this to be a viable option. Although third parties exist, for cases like the United States, those are hardly outside options in presidential elections. Voters decided between the Democrat and Republican. That outside option introduces another layer of complexity that is avoided in the current paper. Also, instead of needing to invoke Reny (1999) to prove existence of an equilibrium, the current model is simple enough that I construct the equilibrium. Au and Kawai go to a level of generality beyond the scope of this simple voting model. In addition, the discussion in Section 4 with more than two types goes beyond their focus, which is mostly about the two type case.

Two other papers study persuasion in a voting context and are closely related: Aköz and Arbatli (2016) and Alonso and Câmara (2016). In Aköz and Arbatli (2016), the voter does not know his preferred position, although the candidates do. This turns the problem into a situation of information manipulation on a single variable, the voter's preferences. That differs from this paper where each political party has private information and tries to persuade voters about that information. This makes more sense when seeing the information

14. Some examples include Gentzkow and Kamenica (2014), Ely, Frankel, and Kamenica (2015), and Ely (2017) for single senders. For multiple senders, like this paper, see Au and Kawai (2016), Gentzkow and Kamenica (2016b, 2017), Koessler, Laclau, and Tomala (2017), and Li and Norman (2017).

revelation as designing a campaign schedule, as I do in this paper. The Republicans choose their schedule. The Democrats choose their schedule. Real campaigns have both elements, but I focus on the schedule design.

Alonso and Câmara (2016) consider a single politician who is trying to get voters to approve certain proposals. It is one sender and a group of receivers. That paper compares different voting rules (say majority vs. unanimity) and compares the outcomes. While that paper and the present both talk of politicians, the papers differ both in the number of senders and the number of receivers, making the problems distinct, though related. An interesting extension would be to analyze different voting rules with competing political parties. There are also many papers that have studied the manipulation of beliefs by politicians outside of elections, such as in Angeletos, Hellwig, and Pavan (2006) and Edmond (2013).

However, most of the voting literature is not touched in this paper. To isolate the role of persuasion, the model simplifies the decisions by parties and voters. Since the political parties can only use persuasion, parties' strategies are extremely limited. There is no strategic picking of who runs, such as in Besley and Coate (1997), or what policy to run on. Therefore, there is no median voter theorem (Downs 1957). However, this lack of convergence to the median voter is not because of any policy preferences by the parties (Wittman 1977, 1983) or because of signaling (Bernhardt, Duggan, and Squintani 2009; Kartik and Mcafee 2007). To simplify the voter's decision, more extensions were left out. Since there is only one voter, she is basically non-strategic. She votes mechanically, given her beliefs. This avoids complications from insincere voting, such as in Austen-Smith and Banks (1996), or strategic abstention, such as in Feddersen and Pesendorfer (1996).

Related to the role of information in an election, this model abstracts from issues related to costly information acquisition (Persico 2004; Gerardi and Yariv 2008; Gershkov and Szentes 2009; Tyson 2016) and the paradoxes that are involved with that (Martinelli 2006). All these simplifications provide possible voting extensions to the growing Bayesian persuasion literature.

6 Conclusion

This paper studies a model where political parties compete solely through information. Uncertainty about the quality of the candidates opens up the possibility for persuasion. Both parties engage in persuasion in order to win or equivalently prevent the other party from winning. As is common in the Bayesian persuasion literature, a sender benefits if he can concavify the value of the receiver's beliefs. In order to not let the other party concavify, each party chooses a uniform distribution of posteriors, which is already concave. Each party sets up its campaign schedule so that the other party cannot take easy advantage of it. This means each party chooses a campaign schedule that induces a uniform distribution of beliefs. The uniform strategy is a shared feature of many voting games. In a persuasion model, the uniform strategy means that the political parties introduce the maximum amount of uncertainty into the election.

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