Price Competition and the Use of Consumer Data

Brian C. Albrecht*

August 11, 2020

Firms have access to vast amounts of data on consumers, which allows them to strategically vary prices across consumers, *i.e.* price discriminate. To study the effects of this data on consumer welfare, I develop a Bertrand duopoly model where each consumer's valuation for each firm's good is uncertain. Instead of imposing that firms have access to specific data, I allow for general information structures; firms may vary in the quality and form of their data. Fixing the available data, due to the discontinuities in Bertrand competition, the unique equilibrium is only supported through price dispersion. I directly construct the unique equilibrium by harnessing features of each firm's residual demand curve. In equilibrium, each firm randomizes her price and generates a unit-elastic residual demand for the other firm. I then vary the available data and compare the welfare consequences. In the baseline model, contrary to common concerns regarding price discrimination derived from the monopoly case, under competition, completely public consumer data (perfect price discrimination) is *optimal* for consumers.

JEL-Classification: D11, D43, D82, D83, L13

Keywords: Bertrand competition, price dispersion, market segmentation

---

Technological change has given both consumers and firms access to information that was previously unimaginable. Consumers can access more data about firms and their products through online reviews. firms can access more data about consumers through deeper market research and tracking data on previous purchases, web searches, or social media posts. That consumer data is vital to a firm's ability to price discriminate.

At the same time, technological change has allowed firms to differentiate products. Identical goods from the supply side (same marginal cost) are not identical from the demand side (different marginal valuation). Gone are the days of one-size-fits-all goods (Neiman and Vavra 2019) To me, most beer tastes the exact same and is substitutable. To

*Coles College of Business, Kennesaw State University. Email: mail@briancalbrecht.com. Thanks to Dirk Bergemann, Keler Marku, Kevin M. Murphy, Sergio Ocampo-Diaz, and David Rahman for helpful comments and conversations. First online draft: June 12, 2018.

others, that claim is preposterous. But all beer companies need to pick prices with both types of consumers in mind. Crucially though, the degree of substitutability, and thus competition, depends on the market make-up and what firms know about consumers. This paper asks, first, how do firms compete in a market where people differ in the degree of substitutability? Second, how does firm access to data on consumers affect how they compete? Finally, when does access to consumer data come back to help or hurt consumers?

To be more specific about the model, I study a stylized model where a continuum of consumers can buy a single good from one of two firms. The two firms compete *à la* Bertrand. However, contrary to a textbook Bertrand model with identical goods—as in Mas-Colell, Winston, and Green (1995, 388) or Tirole (1988, 310)—the consumer may have a different valuation for the goods from the two firms. A firm faces a trade-off between raising the price when the consumer prefers its good and lowering price to compete whenever the consumer prefers the other firm's good. Therefore, I extend a standard model of price discrimination to allow for general, varied access to consumer data by firms.

The paper first considers a simple model to capture the trade-off facing firms where the consumer may be one of three types: one type loyal to each firm for whom the firms' goods are not substitutes and each firm would be a monopolist if they knew the loyal consumers and a third type for whom the firms' goods are perfect substitutes. That setup captures the fundamental trade-off. Each firm wants to raise prices on the consumers loyal to them since they are monopolists. However, for the consumers that are not loyal, the force of Bertrand competition pushes the price down.

In this environment, learning serves two purposes. First, as in the standard monopoly case, learning about customers and being able to use price discrimination allows firms to charge a high price to their loyal consumers and extract more surplus. This hurts consumers. At the same time, a firm can learn which customers they can lure away from the other firm. They learn when it is worth competition on price. In that case, consumer data encourages more intense competition and drives down the price. The model sets up a horse race between the two forces.

My first contribution is methodological. Constructing equilibria is not obvious in this environment. In general, to balance this trade-off, the unique equilibrium involves each firm randomizing over prices. Due to the discontinuity inherent in Bertrand competition, the randomization allows each firm to avoid losing the consumer by only a small amount

on price. This leads to price dispersion in equilibrium.

To construct the unique equilibrium, I use simple observations about the residual demand curve facing each firm and what each firm can guarantee itself by only pricing for loyal consumers.[1] The unique equilibrium on the game involves both firms choosing their price to generate a unit-elastic demand curve for the other firm, making the firms indifferent over a range.

I then ask in a stylized model, given the equilibrium behavior under competition, when is access to consumer data helpful or harmful to consumers and producers? Without competition, the welfare implications of consumer data are clear; information that allows price discrimination weakly raises total profit. This is simply a manifestation of Blackwell (1951, 1953) that information is always valuable for single receiver.[2] Unless output increases, price discrimination lowers consumer surplus.

To study the role of information, I fix underlying valuations, i.e. demand curve in the aggregate market. I then vary the consumer data available to each firm, allowing price discrimination. However, I allow for the case where firms do not have the same information. Therefore, higher-order beliefs matter, e.g. Target doesn't know my exact address, *but* Target knows that Amazon knows... These beliefs affect equilibrium prices.

Finally, given the competitive pricing of the two firms, what is the consumer-optimal form of consumer data? Even in the simple example, the solution to this trade-off is not obvious. However, I show that, contrary to the monopoly case, giving firms *complete information* is consumer optimal, even in an efficient environment where trade always occurs. The increase in consumer surplus does not come from any risk-aversion or increased consumption, as in models where price discrimination increases consumer surplus by increasing trade. Even environments where production does not increase, consumer surplus is maximized under complete information, because the firms to compete more fiercely and charge a lower price when the consumer learns the goods are substitutes for him. The gain from fierce Bertrand competition when the firm's goods are perfect substitutes outweighs the loss when the consumer learns she only values one good and thus that firm can raise its price.

1. The construction of the demand curve is similar to Albrecht (2018)'s analysis of Roesler and Szentes (2017).

2. See Bergemann, Brooks, and Morris (2015) for the general information case of a monopolist.

1 Price Discrimination Literature

This paper contributes to the literature on the relationship between price discrimination and competition is a standard issue in industrial organization–in classic form in Pigou (1920) and Robinson (1933), and in modern form in Borenstein (1985) and Holmes (1989) (See Stole (2007) for a summary).[3] One particular branch of the price discrimination literature shows the optimality of constant-elasticity demand curves, which also show up in my model. This shows up in more standard monopoly models, such as Aguirre (2008), Aguirre and Cowan (2015), Roesler and Szentes (2017), and Condorelli and Szentes (2019). Unit-elastic demand also shows up in non-Bayesian monopoly models, such as Neeman 2003; Bergemann and Schlag 2008; Renou and Schlag 2010. These models involve some form of a maximin utility function or minimax regret, which relates to the outcome of my model where one firm can only achieve his maximin revenue and the rest is competed away by the other firm.

The constant-elasticity implies that there can be multiple prices where marginal revenue equals marginal cost. In my model, this leads to price dispersion. Contrary to the classic price discrimination papers of Stigler (1961) and Burdett and Judd (1983), in my model, price dispersion comes from *each firm* setting a random price, such as in Varian (1980). However, the market power that generates the distribution is due to the goods being imperfect substitutes, and thus having some brand loyalty as in Rosenthal (1980), compared to search frictions.

However, since each firm is randomizing over the set of prices, what Menzio and Trachter (2015) aptly call a "game of cat-and-mouse", the equilibrium prices are not only the lowest, consumer-optimal price found in Roesler and Szentes (2017), but a range of prices that the firm is indifferent between.

Also, this paper contributes to the literature on information design. In particular, I consider the range of possible outcomes that could arise for some information structure, as in Bergemann, Brooks, and Morris (2015). In particular, I am interested in the information structures that are consumer-optimal, as in Roesler and Szentes (2017). However, contrary to both papers, I introduce competition explicitly into the model. Competition is between the receivers of information, making it more like the literature on robust mech-

3. This literature has always included a large empirical component. For a recent example, Chandra and Lederman (2018) show that in the Canadian airline market price discrimination raises prices for some passengers and lowers prices for others. This is consistent with my main result, although in my model the average price is lower.

anism design (Bergemann and Morris 2013), and not between the senders, which is the topic of much of the literature on information design and competition (Boleslavsky and Cotton 2015, 2016; Gentzkow and Kamenica 2016; Li and Norman 2018; Au and Kawai 2020). As in Bergemann, Brooks, and Morris (2015), the consumer-optimal outcome involves extremal markets, where the firms are indifferent among all prices in the support of valuations.

2 Three Type Model

Throughout there are two firms with differentiated goods that cost 0 to produce. For now, we consider a model with three types of consumers with a total measure of one. First, some consumers are loyal to firm 1 and only buy from firm 1. Second, other consumers are loyal to firm 2. Third, the remaining consumers are indifferent and buy from whoever has the cheapest price.[4] Each consumer has unit demand at $p = 1$ from the firm(s) that she is willing to buy from. A consumer's type is a pair of valuations (willingness to pay) $v = (v_1, v_2) \in \{(1,0), (0,1), (1,1)\} = V$. The firms start with a common prior on the market: (m_{10}, m_{01}, m_{11}). In general, we will be interested "interior" markets when no element is zero and all types exist in the economy with positive measure.

A market for a firm i is the collection of consumers that a firm cannot differentiate and must set the same price for. Mathematically, a market is defined as a distribution of the types of consumers. Graphically, with three-types, a market is a point on the unit simplex. For all illustrations, I will consider a starting, aggregate market: $\left(\frac{1}{4}, \frac{1}{6}, \frac{7}{12}\right)$. The aggregate market with no price discrimination is represented by the orange circle in Figure 1.

If both firms have complete data on consumers and know each consumer's type, the firms can segment consumers into three different markets and set a different price for the three consumers. A different price for each type is perfect price discrimination. For the example, complete information means that for each firm there is one market $(1,0,0)$ with measure $\frac{1}{4}$, another market $(0,1,0)$ with measure $\frac{1}{6}$, and a final market $(0,0,1)$ with measure $\frac{7}{12}$. These three markets are represented by the blue diamonds in Figure 1.

No price discrimination and perfect price discrimination are extreme cases. There are other types of markets. Consider a situation where each firm receives a signal that with

4. While I model consumers as having different preferences over sellers' goods, the baseline model could be interpreted as one where consumers differ in whether or not they are "aware" of 1 or 2 firms (Guthmann 2019a, 2019b; Guthmann and Albrecht 2020), or "consider" (Armstrong and Vickers 2020), or are "connected" in a network (Talamàs 2019) or are "captive" (Armstrong and Vickers 2019; Elliott and Galeotti 2019).

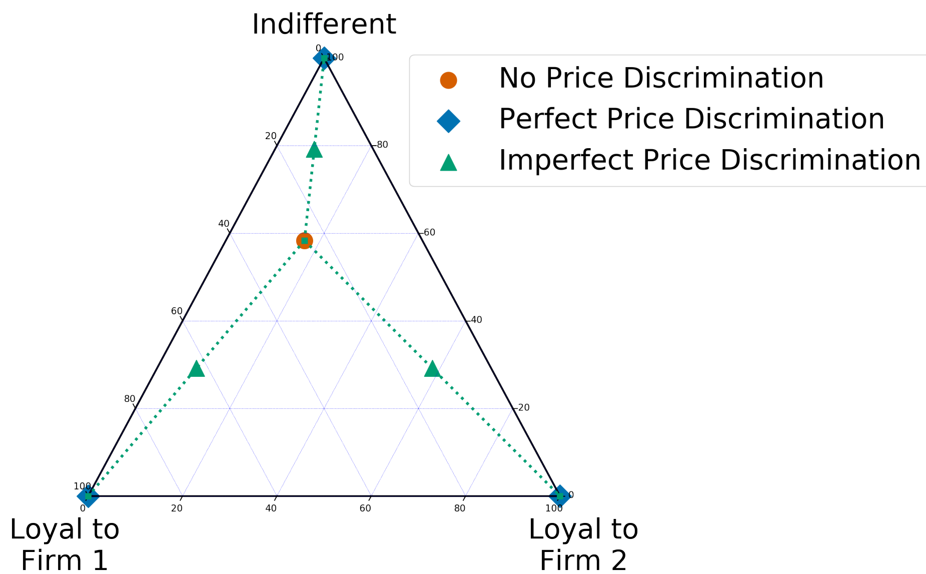


Figure 1: Feasible Markets

probability q correctly reveals a consumer's type and with probability $(1-q)$ the signal comes from a random consumer. This nests the two previous cases with $q = 1$ being a perfect signal and $q = 0$ being an uninformative signal. For any intermediate q, upon seeing a signal that says "this consumer is an indifferent type" the firm properly updates and puts more weight that the consumer being indifferent. With this set of three possible signals, there are three markets: one corresponding to each signal. On the simplex, each feasible market corresponds to a convex combination of the aggregate market and the perfectly discriminated markets. The three markets are plotted by the green triangles in Figure 1.

Formally, the firms' access to consumer data, also called their information structure, is a set of signals for each firm S_i, and a probability distribution which maps the profile of the consumer's values to the profile of signals: $\pi : V \to \Delta(S)$. The utility functions and the information structure (S, π) are the parameters for a game of incomplete information. We will define the rest of the game after fixing (S, π).

Each firm i observes a signal $s_i \in S_i$. A pure strategy for firm i is a price for each signal $\{p_i\}_{s_i} \in \mathbb{R}_+^{|S_i|}$. The discontinuity of payoffs requires that we work with mixed prices for each firm, sometimes called price dispersion. A mixed strategy, $F_i(p|s_i)$, is the probability that $p_i \le p$ given receiving a signal s_i. Let $f_i(p|s_i)$ be the density associated with F_i, when it is defined.

For a given (S, π), a strategy profile is a Bayes Nash equilibrium (BNE) if $f_i(p_i|s_i)$ is

not defined (*i.e.* p_i is played with positive probability) or $f_i(p_i|s_i) > 0$ implies

$$p_i \in \arg\max_{p_i'} \quad p_i' \underbrace{\mathbb{E}[v_i = 1, v_j = 0|s_i]}_{\text{Loyal Customers}} + p_i' \underbrace{\mathbb{E}\left[\left(1 - F_j(p_i)\right), v_i = 1, v_j = 1|s_i\right]}_{\text{Indifferent Customers}},$$

given $F_j(p)$, for all s_i, s_j, i, j. Because we are interested in the effect of information on equilibria, we will be doing comparative statics with respect to the information structure. Also, we will want to ask if there exists any information structure that generates an equilibrium outcome. For that, the following definition is helpful: a strategy profile is a Bayes correlated equilibrium (BCE) if it is a BNE for some information structure (Bergemann and Morris 2016). When looking for the highest possible consumer surplus, the problem requires searching over the set of Bayes correlated equilibria.

2.1 Solving for Equilibrium, Public Consumer Data

Before looking at the set of possible information structures, it is helpful to characterize the equilibrium for particular information structures. First, we will consider public information where both firms have the same data on consumers. The easiest case is complete information, where firms receive a signal that is perfectly reveals the consumers type: $s_1 = s_2 = v$. If the consumer will not buy from the competitor, $v_i = 0$, firm j sets the monopoly price of 1. If the consumer values both goods, $v_1 = v_2 = 1$, because of Bertrand competitive, the price of driven to 0. Combined, the expected price is $m_{10} + m_{01}$. If we plot the distribution of prices the expected price is the area above price distribution.

The equilibrium for the aggregate market, with no additional consumer data and therefore no price discrimination, is more complicated to solve for but has a known solution (Narasimhan 1988).[5] Lemma 1 characterizes the equilibrium.

Lemma 1. *Let $m_{10} \geq m_{01}$. The unique BNE profit is m_{10} for firm 1 and $\frac{m_{10}}{m_{10}+m_{11}}(1 - m_{10})$ for firm 2. The unique strategies are given by*

$$F_1^*(p) = \begin{cases} 0 & p < \underline{p} \\ 1 - \dfrac{\underline{p}(m_{11} + m_{01}) - pm_{01}}{pm_{11}} & p \in \left[\underline{p}, 1\right) \\ 1 & p \geq 1 \end{cases}$$

5. Thanks to Mark Armstrong for this reference and making me aware of the related work of Armstrong and Vickers (2019).

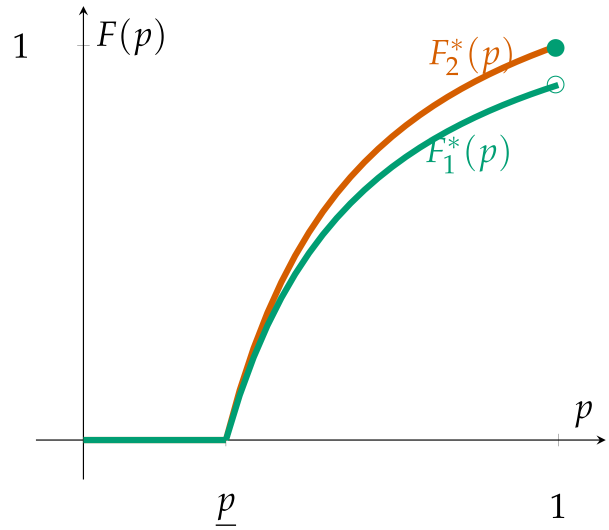


Figure 2: Equilibrium Price Distribution

$$F_2^*(p) = \begin{cases} 0 & p < \underline{p} \\ 1 - \dfrac{m_{10}(1-p)}{pm_{11}} & p \in \left[\underline{p}, 1\right], \end{cases}$$

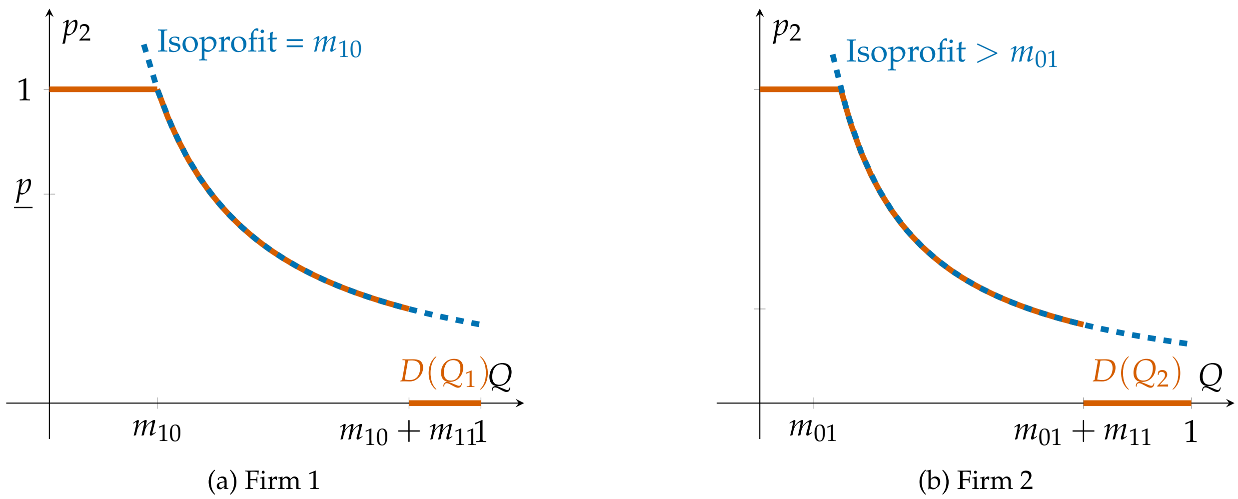
Proof. Despite the result appearing before in Narasimhan (1988), I present a new proof here for completeness and because the proof technique is used in the extensions. First, let us plot the proposed equilibrium distribution of prices in Figure 2. Conditional on $v_1 = 1$, firm 1 assigns probability $\frac{m_{10}}{m_{10}+m_{11}}$ to being the monopolist. Regardless of what firm 2 does, firm 1 will never set a price below $\underline{p} = \frac{m_{10}}{m_{10}+m_{11}}$. Therefore, neither will firm 2.

The rest of the construction relies on simple observations of each firm's best-response when facing a residual demand curve. Translating demand curves into game-theoretic terms, best-responding by each seller means choosing a price and quantity subject to the constraint that the pair is on her demand curve.[6] To find the best-response, plot firm 1's indifference curves, which are just the iso-revenue curves, since the cost is zero.[7] This is also the worst-case (maximin) profit for firm 1 since he could never do worse than if $p_2 = 0$.

To show this strategy is an equilibrium, in orange in Figure 3b, I plot firm 1's residual

6. The best-response could also be found by constructing the marginal revenue (MR) curve. However, because MR is discontinuous, the optimal expected quantity is the maximum quantity such that MR > MC = 0, instead of the standard MR=MC condition used for continuous functions.

7. I thank Kevin M. Murphy for pointing out the benefits of plotting indifference curves in Marshallian, quantity-price coordinates.



(a) Firm 1

(b) Firm 2

Figure 3: Equilibrium Residual Demand

demand, given $F_2^*(p)$, $D(Q_1)$. Notice that to the right of m_{10}, where the residual demand comes from the indifferent buyer, the curve is simply one minus the $F_2^*(p)$ of firm 2's randomized price. The only complication is when a specific price is set with positive probability, which will be addressed shortly. Firm 1's problem is to maximize profits, subject to its residual demand curve. Because the marginal cost is zero, the isoprofit curves are simply given by $\pi = p_1 Q_1$.

Because of the shape of the residual demand curve, firm 1 is indifferent between any price between $\frac{m_{10}}{m_{10}+m_{11}}$ and 1. Therefore, firm 1 would be optimally mixing with any price in that range. For any mixing, the highest profit is simply given by m_{10}.

We do the same thing for firm 2 in Figure 3b. Again, firm 1 has picked a strategy. However, this time, since firm 1 set $p = 1$, the top of firm 2's residual demand extends out beyond just it's loyal consumers with measure m_{01}, and firm 2 can still receive some consumers when $p_1 = 1$.

Therefore, we have shown that each strategy is the best-response to the other strategy and constructed an equilibrium. Uniqueness is immediate as well.

■

Due to the discontinuity of payoffs in Bertrand competition, if $p \neq 1$, every equilibrium involves a distribution of prices. For either firm to be indifferent, $p \times Q =$ constant. The distribution of prices is, therefore, proportional to $-\frac{1}{p}$. The only exception is that there could be a possible mass point at $p = 1$. This distribution will show up in any equilibrium.

2.2 Construction of Mixed Strategy Equilibrium

This subsection constructs the equilibrium price distribution of Lemma 1, as compared to just verifying that the given strategy is an equilibrium. This is worthwhile because the construction can be applied to a wide range of markets. Translating demand curves into game-theoretic terms, best-responding by each seller means choosing a price and quantity subject to the constraint that the pair is on her demand curve. As a first guess of the equilibrium strategies, let us first find firm 1's maximin strategy; suppose that $p_2 = 0$. The best-response price is one and the expected quantity that firm 1 sells is m_{10}.[8]

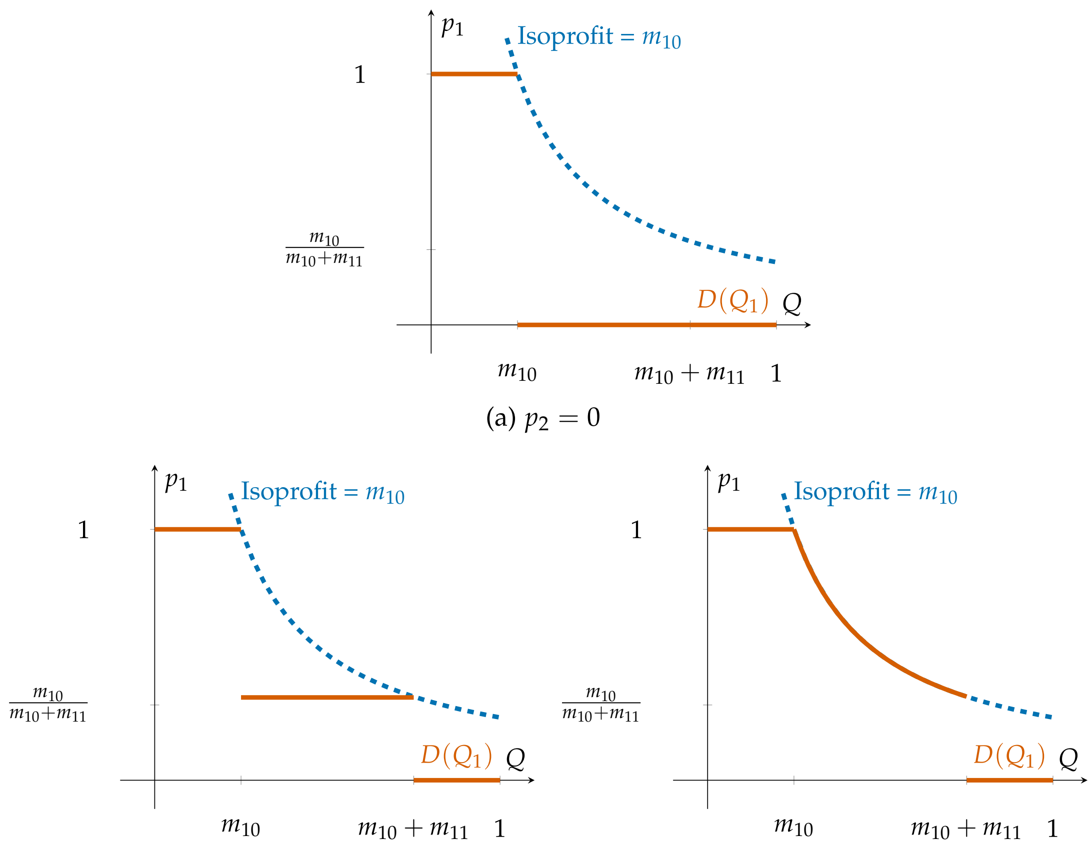
However, setting a price of zero is not optimal for firm 2. Now I ask, what is the highest (pure strategy) price that firm 2 can charge without changing firm 1's behavior? That case is a better-reply, even if it is not a best-reply. Increasing the price of firm 2 increases firm 1's demand curve over the range $(m_{10}, m_{10} + m_{11})$ (selling to the indifferent types), by p_2. By plotting firm 1's iso-profit curve, we can see that firm 2's price can be increased to $\frac{m_{10}}{m_{10}+m_{11}}$ without inducing firm 1 to change. While this is not going to be an equilibrium, it will allow us to bound prices. First, it shows that it is never a best-response for firm 2 to set a price lower than $\underline{p}$. She can raise her price to $\underline{p}$ while one is still best-responding with her price of one. It also creates a lower bound for firm 1's price. Even if firm 1 sold every time the buyer was not loyal to the firm 2, $Q_1 = m_{10} + m_{11}$, she would still make less profit for any price below $\underline{p}$ than if she just set a price of one. Therefore, both sellers will never drop their price below $\underline{p}$ in any equilibrium.

Firm 2 can still do better by randomizing between $\underline{p}$ and one. This is like setting a non-linear price and induces a residual demand curve for firm 1 that is no longer flat in the middle.[9] The demand curve varies with the cumulative distribution of firm 2's randomized strategy. Again, the demand curve for firm 1 is the price set by firm 2, but now in a probabilistic sense. The residual demand curve for the indifferent buyer is simply one minus the CDF of firm 2's randomized price.

The only complication is a specific price is set with positive probability. Better-replying by firm 2 with randomization means increasing firm 1's demand curve for all prices between $\underline{p}$ and one such that firm 1 is still indifferent. Figure 4c shows the demand curve where firm 1 is indifferent between a range of prices, which is generated by a particu-

8. The best-response could also be found by constructing the marginal revenue (MR) curve. However, because MR is discontinuous, the optimal expected quantity is the maximum quantity such that MR > MC = 0, instead of the standard MR=MC condition used for continuous functions.

9. Using the indifference curve to trace out the optimal policy resembles one construction of the optimal income tax rate in an optimal income tax model, like Mirrlees (1971).

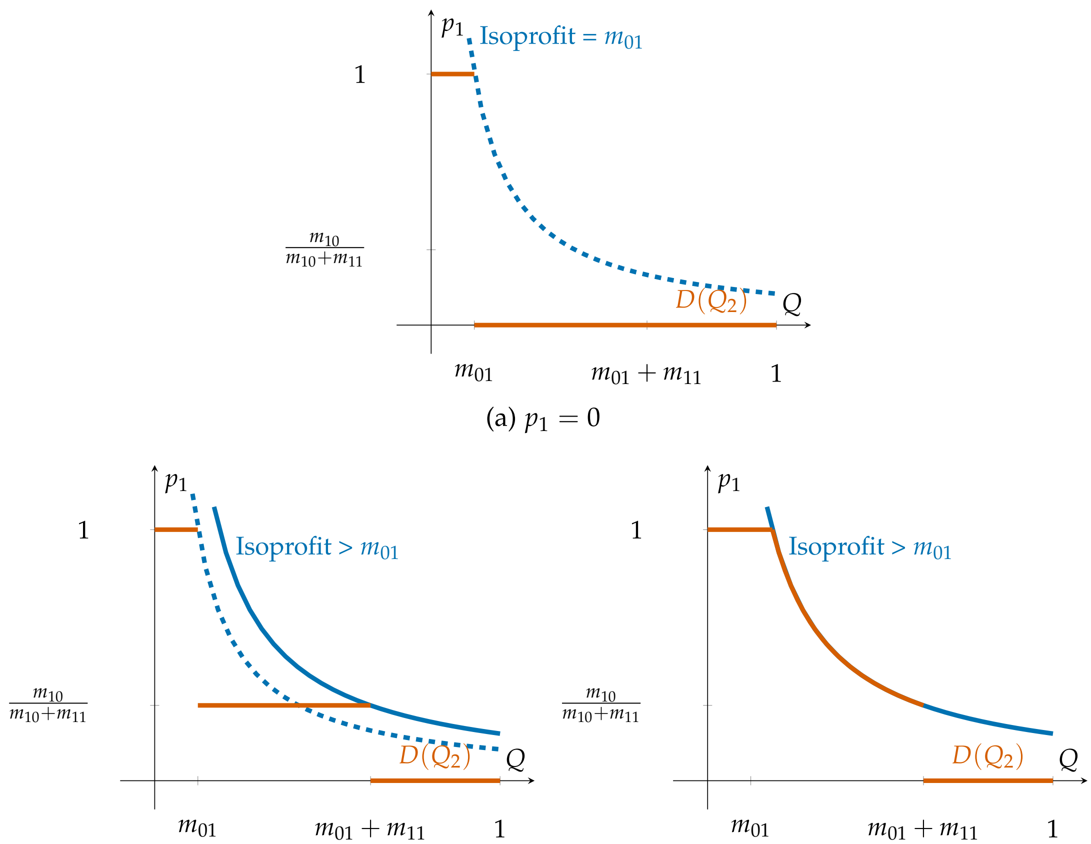


(a) $p_2 = 0$

(b) $p_2 = \underline{p}$

(c) $F_2^*(p)$

Figure 4: Firm 1's Residual Demand and Indifference Curves



(a) $p_1 = 0$

(b) $p_1 = \underline{p}$

(c) $F_1^*(p)$

Figure 5: Firm 2's Residual Demand and Indifference Curves

lar pricing strategy. This pricing strategy is also firm 2's equilibrium pricing strategy, as explained in Lemma 1.[10]

However, even with all of this mixing, neither can achieve more in equilibrium than winning their respective market with $\underline{p}$. The randomization protects against undercutting by the other seller but in equilibrium does not generate any additional profit, resembling the normal Bertrand force that drives profits to zero.[11]

This process of finding the mixed strategy is standard. We search for the strategy of firm 1 that makes firm 2 indifferent (willing to mix) and vice versa. But we are looking for a mixed strategy over a continuum, so it is not immediately easy to visually. That is where the residual demand and iso-profit curves come in. The demand curves immediately allow the economist to picture a continuous variable easily.

2.3 Consumer and Firm Optimal Public Data

It is worth pointing out a corollary of the above proposition: no matter the distribution of types, complete data on consumers is always better than no data on consumers.

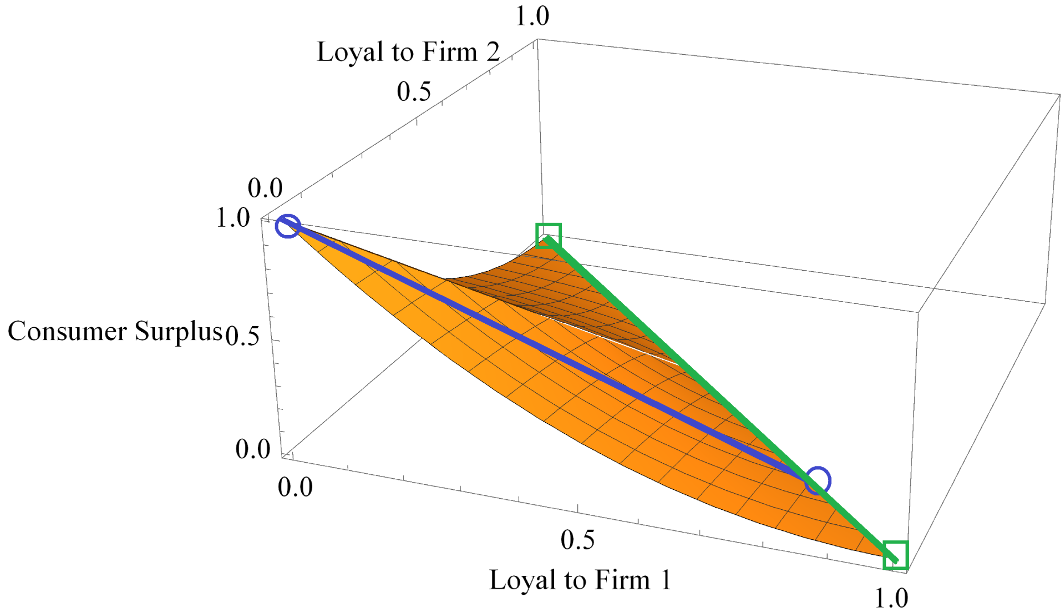
Corollary 1. *1) Consumer surplus under perfect price discrimination is weakly higher than consumer surplus under no price discrimination. 2) Total profits under perfect price discrimination are weakly lower than total profits under no price discrimination. 3) The relationships are strict if* $m_{10} \neq m_{01}$.

Notice that we have already done the work to solve the general model of public information. The fact that this is the "aggregate" market is not important. The equilibrium price distributions will be the same if the market was the result of some consumer data signals. Therefore, we can immediately plot the consumer surplus for any market in the simplex, as shown in Figure 6a, or the expected price as shown in Figure 6b

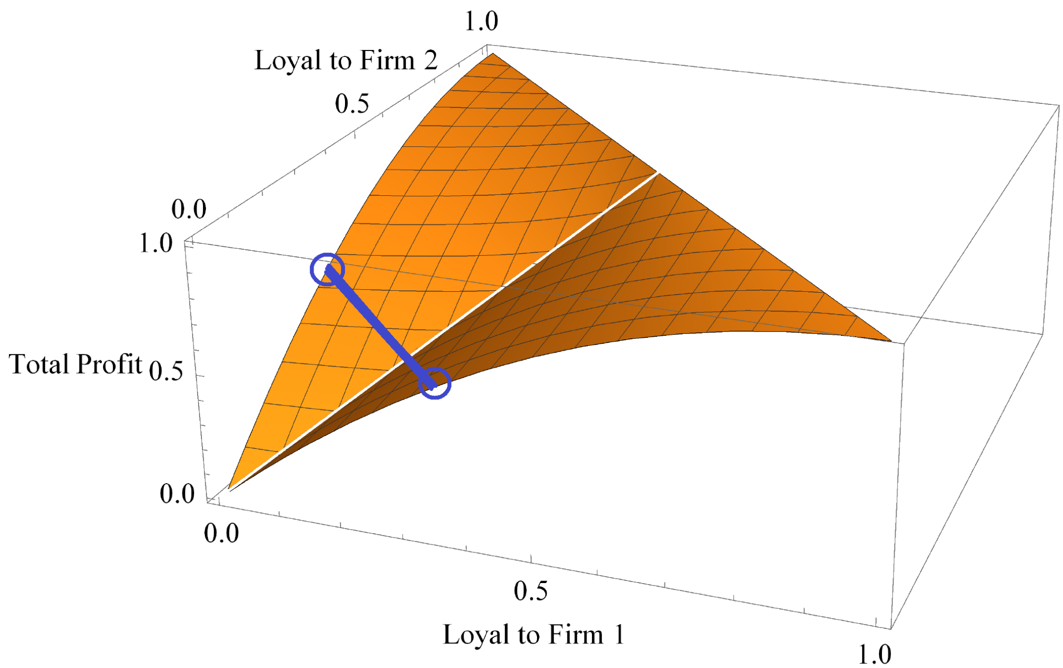
In addition, we can search for the buyer optimal (lowest expected price) and seller optimal (highest expected price) information structures. Proposition 1 proves that complete information is consumer optimal.

10. If the parties are symmetric ($m_{01} = m_{10}$), each seller receives her loyal customers and we are back to perfect competition, where each person receives her marginal product (Ostroy 1984), as is standard for symmetric Bertrand competition.

11. This also resembles matching pennies or the information design model in Albrecht (2017), where there is political competition with discontinuities and randomization prevents the other party from winning. However, randomization does little to the odds of winning in equilibrium. Both matching pennies and political competition are zero-sum games. With this is technically not zero-sum, it shares some characteristics.



(a) Consumer Surplus



(b) Total Profit

Proposition 1. *With only public data, consumer surplus is maximized under perfect price discrimination.*

Proof. Bayes' Law only restricts the ex-post distributions of types to sum to the prior. Therefore, we can maximize/minimize over all feasible distributions (markets) by examining the "concavification" (Kamenica and Gentzkow 2011) of the value function of interest: consumer surplus.

The profit minimizing, and therefore consumer surplus maximizing, is given by the concave envelope on the function. Because the total profit is convex on each half of the simplex, the concave envelope touches the convex function at extreme point, with one of the split markets being just indifferent consumers, $(0, 0, 0)$ and one of the markets being the combination of loyal customers $(\frac{m_{10}}{m_{10}+m_{01}}, \frac{m_{01}}{m_{10}+m_{01}}, 0)$. The splitting is given by the blue line and circles. However, because firm 1 can ignore firm 2's loyal customers, that market can be further split according to the green line and squares, so that the effective profit-maximizing markets are complete information: $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$.

Therefore, we have shown that complete information is optimal for consumers. ■

The total profit-maximizing data structure can similarly be found by concavification. First, let us follow Armstrong and Vickers (2019, 2020), and define a *nested market* as a market where one firm's potential customers are a subset of the other firm's. On the simplex, the set of nested markets are the two edges: $(x, 0, 1 - x)$ and $(0, y, 1 - y)$. We then have the following proposition:

Proposition 2. *With only public data, total profits are maximized with nested markets.*

Proof. The proof is immediate from concavification and shown by Figure 6b as the blue line and circles. ■

Everything so far has involved public data on consumers. The next section extends the model to allow for private information by each firm.

3 Private Consumer Data

Most models of market competition assume firms have public information, as above. However, increases in data collection make this assumption less tenable. I am a different person according to Amazon vs. Target. Amazon knows that I am a South Minneapolis

male who buys too many economics books. Target only knows that I am a person shopping in Minneapolis on Tuesday. Still, the firms' pricing strategies will be intertwined and part of an equilibrium; Target still needs to consider Amazon's pricing.

To understand this type of asymmetric consumer data, let us consider a simple case where firm 1 has complete information and firm 2 has only aggregate information. Again, each market for a firm is a point on the simplex. However, now higher-order beliefs matter. There is no longer an objective "market", just overlapping markets. Firm 2 knows that firm 1 knows the true type. Firm 2 only needs to consider competition from firm 1 when the true type is indifferent. For pricing, firm 2's relevant market is made of its loyal consumers and indifferent. The markets are plotted in Figure 6.

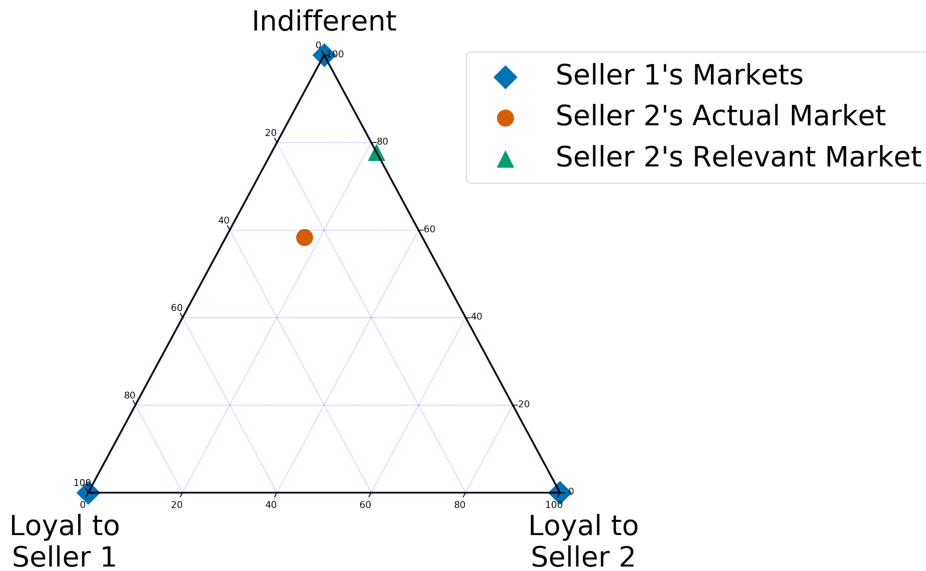


Figure 6: Overlapping Markets

Given the profit functions characterized in Lemma 1, it is easy to show that, relative to no consumer data, when firm i has access to consumer data that allows perfect price discrimination, firm i's profit strictly increases, firm j's profit strictly decreases, total profit can either increase or decrease. However, if the firms are symmetric ($m_{10} = m_{01}$), then profits increase and consumer surplus decreases.

Proposition 3. *For any symmetric interior market, perfect price discrimination by only one firm strictly decreases consumer surplus relative to no price discrimination.*

Comparing Proposition 3 to Corollary 1 highlights the interplay of competition and information. The powers of competition to drive down consumer prices are harnessed

when both firms have access to information. When only one firm has access, the firm uses that information for rent-extraction as in the monopoly case.

But that is still a very simple information structure. In general, overlapping markets can be much more complex and involve correlated information, where the signal that firm i receives is correlated with the signal that firm j receives. The following proposition shows that the worst case for consumers is imperfectly correlated data, which effectively allows the firms to collude and avoid Bertrand competition.

Proposition 4. *Imperfectly correlated data can strictly increase prices relative to any public information structure.*

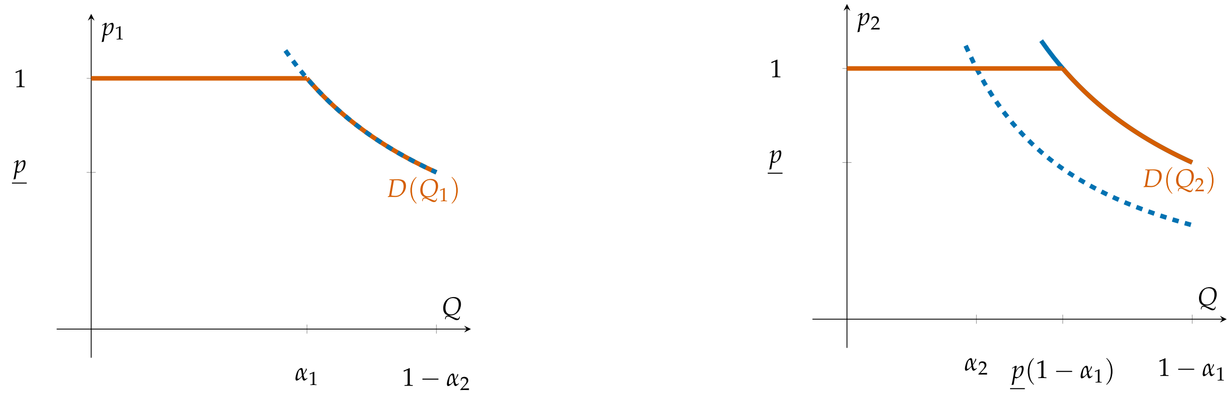
Proof. Imagine an information designer[12] who reveals consumer data to the firms and recommends an incentive-compatible price. The designer commits the following information for the indifferent consumers, only partially revealing the indifferent consumers.

<table>
<tr><td></td><td></td><td colspan="2">Signal to Firm 2</td></tr>
<tr><td></td><td></td><td>Reveal</td><td>Do Not</td></tr>
<tr><td>Signal to Firm 1</td><td>Reveal</td><td>$1 - \alpha_1 - \alpha_2$</td><td>α_1</td></tr>
<tr><td></td><td>Do Not</td><td>α_2</td><td>0</td></tr>
</table>

For now, fix $\alpha_1 \geq \alpha_2$ and assume that the designer recommends $p = 1$ when a firm sees no signal and recommends a corresponding mixed distribution when it is revealed the customer is indifferent. We will construct an equilibrium that increases profit relative to any public signal.

Consider the case when firm 1 receives a signal and therefore knows the consumer is indifferent. Even though firm 1 knows the consumer is indifferent, prices will not be driven to zero because firm 1 does not know whether firm 2 knows that the consumer is indifferent. With probability α_1, firm 2 did not receive a signal and will, therefore, follow the designer's recommend strategy and set a price of 1. Otherwise, firm 2 will price according to a distribution over $[\underline{p}, 1]$. How high can the information designer raise firm 2's while still keeping firm 1's behavior incentive compatible? Just as Figure 3a, firm 2 can price according to a distribution that makes firm 1 indifferent between setting any price between firm 2's lowest price $\underline{p}$ and 1.

12. I stick to the term information designer to highlight the similarities to questions in the information design literature (Bergemann and Morris 2019), but this could also be thought of as mediator, as is used in other models of collusion, *e.g.* Rahman and Obara (2010) and Rahman (2014).



(a) Firm 1

(b) Firm 2

Figure 7: Residual Demand after Signal

We can do the same thing for when firm 2 receives a signal. However, because firm 1 is only willing to reduce the price to $\underline{p}$, firm 2 never needs to price any lower. This means that Firm 2 can receive a higher profit than his maximin. This is shown in Figure 7b and mirrors the public information case shown in Figure 5b.

Now that we have calculated $\underline{p}$ and the best-response when each firm receives a signal, we only need to verify that setting a price of 1 is a best-response when each firm does not receive a signal. First, consider the case of firm 1 when he sees no signal. That could be because the consumer is loyal to firm 1 (which happens with probability m_{10}) or because the information simply was not revealed (which happens with probability $\alpha_2 m_{11}$). The residual demand when the firm does not receive a signal is graphed in Figure 8. This exact distribution depends on the parameter α_2. We have therefore constructed the strategies so that it is a best-response for firm 1 to keep a price of 1 and firm 2 is also maximizing his profit. The same argument goes through for when firm 2 receives a signal.

To review, fixing α, when the firms receive a signal, they must want to set a distribution of prices. The relevant ICs (from Figure 7) are

$$\underline{p} \geq \frac{\alpha_1}{1-\alpha_2} \quad \text{and} \tag{1}$$

$$\underline{p} \geq \frac{\alpha_2}{1-\alpha_1}, \tag{2}$$

where at least the first constraint is binding.

When the firm receives no signal, setting a price of 1 must be a best-response. In principle, we need to worry about deviating to any other price, but for simplicity let us

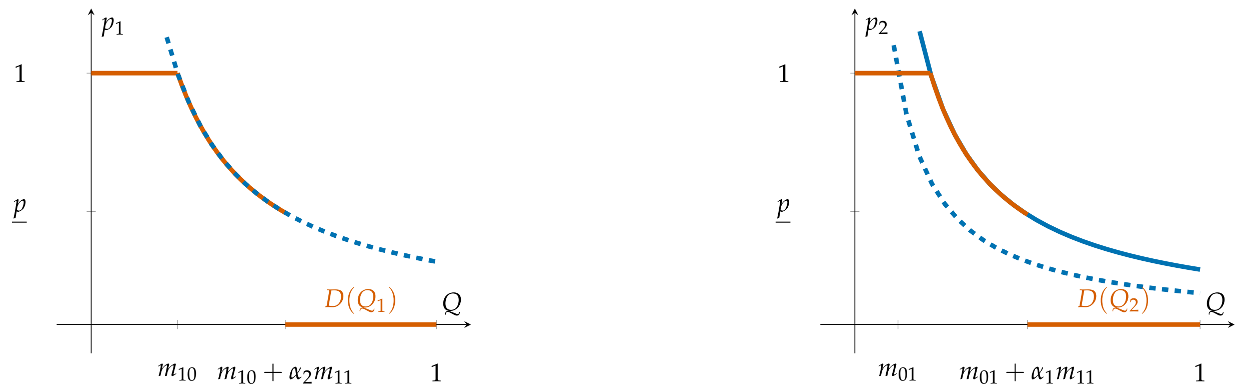


Figure 8: Residual Demand after No Signal

drop the interior prices, knowing the distribution can be constructed to make sure those are satisfied. The two incentive constraints (seen from Figure 8) are therefore

$$m_{10} \geq \underline{p}\left(m_{10} + \alpha_2 m_{11}\right) \quad \text{and} \tag{3}$$

$$m_{01} \geq \underline{p}\left(m_{01} + \alpha_1 m_{11}\right), \tag{4}$$

where at least the first constraint is binding.

Therefore, we have solved for the equilibrium as functions of α_1 and α_2. Now we can simply add the profit from each signal to find the total expected profit/price:

$$\text{Expected Price} = \underbrace{\alpha_1}_{\text{Figure 7a}} + \underbrace{\frac{\alpha_1}{1-\alpha_2}(1-\alpha_1)}_{\text{Figure 7b}} + \underbrace{m_{10}}_{\text{Figure 8a}} + \underbrace{\frac{\alpha_1}{1-\alpha_2}(m_{01} + \alpha_1 m_{11})}_{\text{Figure 8b}}$$

To maximize prices, the information designer's problem is to choose α to maximize profits, subject to the incentive constraints.

■

With all of these different types of consumer data consider, we can plot the distribution of equilibrium prices for the six cases in Figure 9. The expected price and profit is the area above the distribution. Therefore, consumer surplus is the area under the distribution. The vector of surplus for each case is plotted in Figure 10. Again, these are for an aggregate market of $\left(\frac{1}{4}, \frac{1}{6}, \frac{7}{12}\right)$.

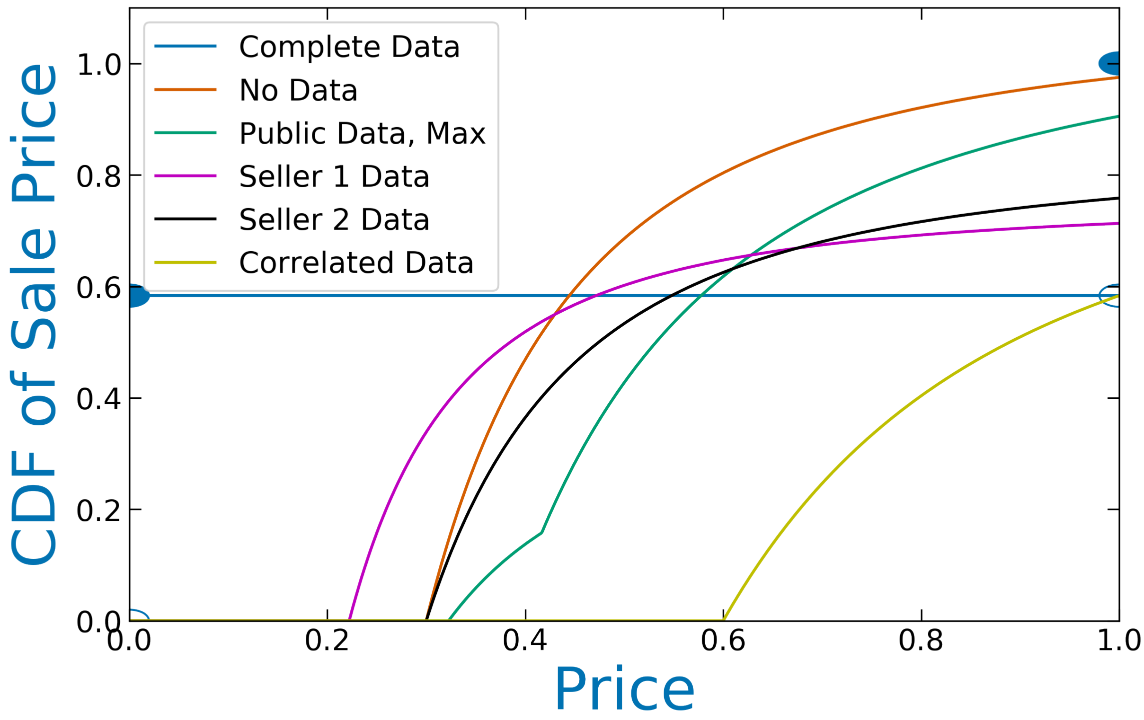


Figure 9: Price Distribution

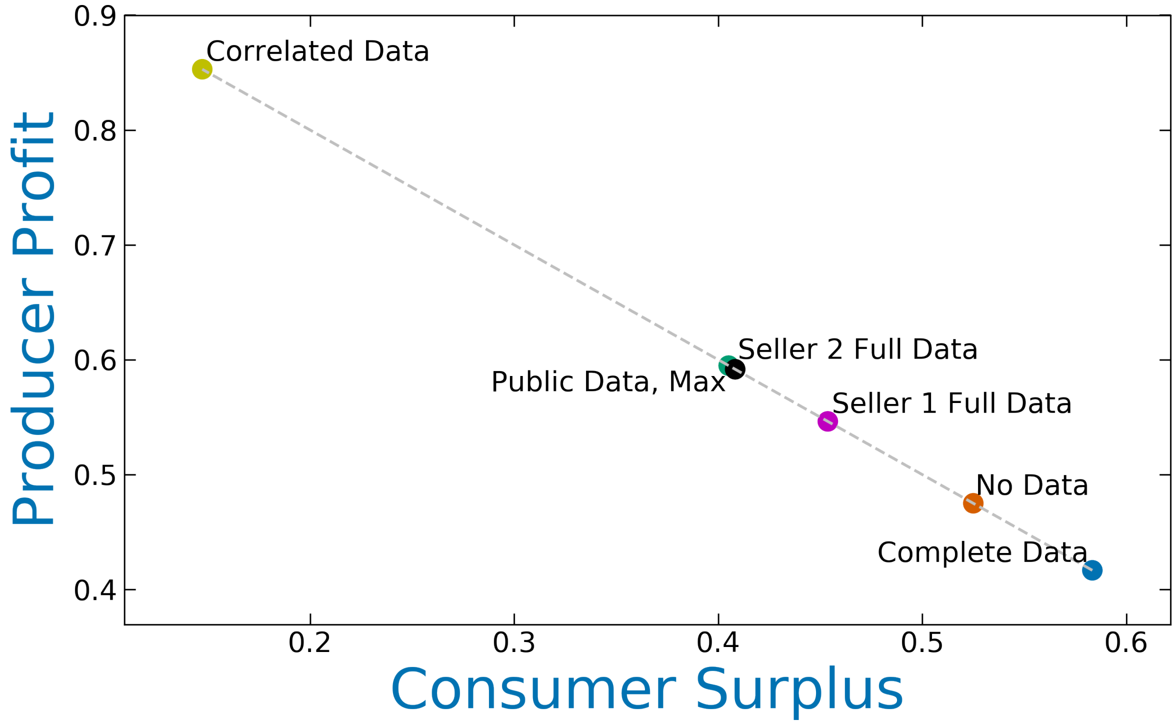


Figure 10: Surplus Division

4 Conclusion

This paper studies a version of Bertrand competition where firms may have access to more detailed consumer data. In equilibrium, each firm chooses a strategy which is a distribution of strategies that induces a unit-elastic residual demand curve for the other firm. This makes each firm indifferent over a range of prices and ensures an equilibrium, given some uncertainty.

I then analyze the welfare consequences of the consumer and duopolists having additional information about the consumer's tastes. With additional information, the firms can use that information to set different pricing strategies, i.e. use third-degree price discrimination. I solve for the consumer-optimal information structure: completely revealing data. While firms can use data to raise its price above marginal cost and thus exploit their market power through price discrimination when the consumer prefers one good, complete information unleashes the power of competition when firms must ruthlessly compete on price. Competition works best in full light.

References

Aguirre, Iñaki. 2008. "Output and misallocation effects in monopolistic third-degree price discrimination." *Economics Bulletin* 4 (11): 1–11. (Cited on page 4).

Aguirre, Iñaki, and Simon G. Cowan. 2015. "Monopoly price discrimination with constant elasticity demand." *Economic Theory Bulletin* 3, no. 2 (October): 329–340. doi:10.1007/s40505-014-0063-3. http://link.springer.com/10.1007/s40505-014-0063-3. (Cited on page 4).

Albrecht, Brian C. 2017. "Political Persuasion." https://briancalbrecht.github.io/albrecht-political-persuasion.pdf. (Cited on page 13).

———. 2018. "Some Price Theory of Buyer-Optimal Learning." (Cited on page 3).

Armstrong, Mark, and John Vickers. 2019. "Discriminating against Captive Customers." *AER: Insights* 1 (3): 257–272. doi:10.1257/aeri.20180581. https://doi.org/10.1257/aeri.20180581. (Cited on pages 5, 7, 15).

———. 2020. "Patterns of Price Competition and the Structure of Consumer Choice." (Cited on pages 5, 15).

Au, Pak Hung, and Keiichi Kawai. 2020. "Competitive information disclosure by multiple senders." *Games and Economic Behavior* 119 (January): 56–78. ISSN: 10902473. doi:10.1016/j.geb.2019.10.002. (Cited on page 5).

Bergemann, Dirk, Benjamin Brooks, and Stephen Morris. 2015. "The Limits of Price Discrimination." *The American Economic Review* 105 (3): 921–957. ISSN: 00028282. doi:10.1257/aer.20130848. http://dx.doi.org/10.1257/aer.20130848. (Cited on pages 3–5).

Bergemann, Dirk, and Stephen Morris. 2013. "Robust Predictions in Games with Incomplete Information." *Econometrica* 81 (4): 1251–1308. ISSN: 0012-9682. doi:10.3982/ECTA11105. http://doi.wiley.com/10.3982/ECTA11105. (Cited on page 5).

———. 2016. "Bayes correlated equilibrium and the comparison of information structures in games." *Theoretical Economics* 11 (2): 487–522. ISSN: 19336837. doi:10.3982/TE1808. http://doi.wiley.com/10.3982/TE1808. (Cited on page 7).

———. 2019. "Information design: A unified perspective." *Journal of Economic Literature* 57, no. 1 (March): 44–95. ISSN: 00220515. doi:10.1257/jel.20181489. (Cited on page 17).

Bergemann, Dirk, and Karl H. Schlag. 2008. "Pricing without Priors." *Journal of the European Economic Association* 63 (2): 560–569. http://www.jstor.org/stable/40282665. (Cited on page 4).

Blackwell, David. 1951. "Comparison of Experiments." In *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability,* edited by Jerzy Neyman, 93–102. Berkeley: University of California Press. (Cited on page 3).

———. 1953. "Equivalent Comparisons of Experiments." *The Annals of Mathematical Statistics* 24 (2): 265–272. doi:10.2307/2236332. http://www.jstor.org/stable/2236332. (Cited on page 3).

Boleslavsky, Raphael, and Christopher Cotton. 2015. "Grading Standards and Education Quality." *American Economic Journal: Microeconomics* 7, no. 2 (May): 248–279. ISSN: 1945-7669. doi:10.1257/mic.20130080. http://pubs.aeaweb.org/doi/10.1257/mic.20130080. (Cited on page 5).

———. 2016. "Limited capacity in project selection: competition through evidence production." *Economic Theory.* doi:10.1007/s00199-016-1021-0. (Cited on page 5).

Borenstein, Severin. 1985. "Price discrimination in free-entry markets." *Rand Journal of Economics* 16 (3): 380–397. (Cited on page 4).

Burdett, Kenneth, and Kenneth L. Judd. 1983. "Equilibrium Price Dispersion." *Econometrica* 51 (4): 955–969. http://www.jstor.org/stable/1912045. (Cited on page 4).

Chandra, Ambarish, and Mara Lederman. 2018. "Revisiting the Relationship between Competition and Price Discrimination." *American Economic Journal: Microeconomics* 10 (2): 190–224. doi:10.1257/mic.20160252. https://doi.org/10.1257/mic.20160252. (Cited on page 4).

Condorelli, Daniele, and Balazs Szentes. 2019. "Information Design in the Hold-up Problem." *Journal of Political Economy.* http://personal.lse.ac.uk/szentes/docs/holdup.pdf. (Cited on page 4).

Elliott, Matthew, and Andrea Galeotti. 2019. "Market segmentation through information." (Cited on page 5).

Gentzkow, Matthew, and Emir Kamenica. 2016. "Competition in Persuasion." *Review of Economic Studies* 84, no. 1 (January): 300–322. ISSN: 1467937X. doi:10.1093/restud/rdw052. (Cited on page 5).

Guthmann, Rafael R. 2019a. "Price Dispersion as a Result of Unawareness." https://ssrn.com/abstract=3290863. (Cited on page 5).

———. 2019b. "Price Dispersion in Dynamic Competition." https://ssrn.com/abstract=3290853. (Cited on page 5).

Guthmann, Rafael R., and Brian C. Albrecht. 2020. "On the Informational Efficiency of Decentralized Price Formation." (Cited on page 5).

Holmes, Thomas J. 1989. "The Effects of Third-Degree Price Discrimination in Oligopoly." *The American Economic Review* 79:244–250. doi:10.2307/1804785. https://www.jstor.org/stable/1804785. (Cited on page 4).

Kamenica, Emir, and Matthew Gentzkow. 2011. "Bayesian Persuasion." *The American Economic Review* 101 (6): 2590–2615. doi:10.1257/aer.101.6.2590. (Cited on page 15).

Li, Fei, and Peter Norman. 2018. "On Bayesian persuasion with multiple senders." *Economics Letters* 170 (September): 66–70. ISSN: 01651765. doi:10.1016/j.econlet.2018.05.023. (Cited on page 5).

Mas-Colell, Andreu, Michael D. Winston, and Jerry R. Green. 1995. *Microeconomic Theory.* 977. New York: Oxford University Press. ISBN: 0-19-510268-1. (Cited on page 2).

Menzio, Guido, and Nicholas Trachter. 2015. "Equilibrium price dispersion with sequential search." *Journal of Economic Theory* 160 (December): 188–215. doi:10.1016/J.JET.2015.09.004. (Cited on page 4).

Mirrlees, James A. 1971. "An Exploration in the Theory of Optimum Income Taxation." *The Review of Economic Studies* 38 (2): 175–208. http://www.jstor.org/stable/2296779. (Cited on page 10).

Narasimhan, Chakravarthi. 1988. "Competitive Promotional Strategies." *The Journal of Business* 61 (4): 427–449. (Cited on pages 7, 8).

Neeman, Zvika. 2003. "The effectiveness of English auctions." *Games and Economic Behavior* 43, no. 2 (May): 214–238. doi:10.1016/S0899-8256(03)00005-8. (Cited on page 4).

Neiman, Brent, and Joseph Vavra. 2019. "The Rise of Niche Consumption." Cambridge, MA, August. doi:10.3386/w26134. http://www.nber.org/papers/w26134.pdf. (Cited on page 1).

Ostroy, Joseph M. 1984. "A Reformulation of the Marginal Productivity Theory of Distribution." *Econometrica* 52 (3): 599–630. (Cited on page 13).

Pigou, Arthur Cecil. 1920. *The Economics of Welfare.* 4th Editio. London: Macmillan. 1932. (Cited on page 4).

Rahman, David. 2014. "The Power of Communication." *The American Economic Review* 104, no. 11 (November): 3737–3751. ISSN: 0002-8282. doi:10.1257/aer.104.11.3737. http://pubs.aeaweb.org/doi/10.1257/aer.104.11.3737. (Cited on page 17).

Rahman, David, and Ichiro Obara. 2010. "Mediated Partnerships." *Econometrica* 78 (1): 285–308. ISSN: 0012-9682. doi:10.3982/ecta6131. (Cited on page 17).

Renou, Ludovic, and Karl H. Schlag. 2010. "Minimax regret and strategic uncertainty." *Journal of Economic Theory* 145, no. 1 (January): 264–286. doi:10.1016/J.JET.2009.07.005. (Cited on page 4).

Robinson, Joan. 1933. *The Economics of Imperfect Competition.* London: Macmillan. (Cited on page 4).

Roesler, Anne-Katrin, and Balazs Szentes. 2017. "Buyer-Optimal Learning and Monopoly Pricing." *The American Economic Review* 107 (7): 2072–2080. doi:10.1257/aer.20160145. (Cited on pages 3, 4).

Rosenthal, Robert W. 1980. "A Model in which an Increase in the Number of Sellers Leads to a Higher Price." *Econometrica* 48, no. 6 (September): 1575. doi:10.2307/1912828. (Cited on page 4).

Stigler, George J. 1961. "The Economics of Information." *Journal of Political Economy* 69 (3): 213–225. (Cited on page 4).

Stole, Lars A. 2007. "Price Discrimination and Competition." Chap. 34 in *Handbook of Industrial Organization,* edited by Mark Amstrong and Rob Porter, 3:2221–2299. Amsterdam: Elsevier. ISBN: 9780444824356. doi:10.1016/S1573-448X(06)03034-2. (Cited on page 4).

Talamàs, Eduard. 2019. "Price dispersion in stationary networked markets." *Games and Economic Behavior* 115:247–264. ISSN: 0899-8256. doi:https://doi.org/10.1016/j.geb.2019.03.005. http://www.sciencedirect.com/science/article/pii/S0899825619300405. (Cited on page 5).

Tirole, Jean. 1988. *The Theory of Industrial Organization.* MIT Press. (Cited on page 2).

Varian, Hal R. 1980. "A Model of Sales." *The American Economic Review* 70 (4): 651–659. http://www.jstor.org/stable/1803562. (Cited on page 4).